Theory and practice of runoff space-time distribution

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Abstract Based on the domestic and foreign concerning researches, this paper submits the runoff space-time distribution theory which shows evident scientific significances and powerful practical functions. On the basis of digital basin unit cell deriving from the digital elevation model (DEM) and assumption of linear confluence, this theory has been applied successfully to the runoff correlation researches in humid regions. In order to prove the adaptability of the theory in arid and semi-drought regions, this paper is used to the runoff correlation analysis in Wuding River basin——a tributary of Yellow River Basin, and has gained preliminary effective verification.

Keywords: runoff; space-time distribution, theory, the Wuding River, correlation coefficient, practice.

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The runoff is the basic object of narrow sense of water resources assessment. The runoff space-time distribution rule is the scientific cornerstone of the research in water resources space-time evolution and variation rules. Theoretical quantitative description on the space-time distribution characters will not only establish theoretical base in the future water resources space-time evolution and variation researches but also guide the practice on the water resources development and utilization.

1 Research development of runoff space-time distribution theory

The runoff space-time distribution is influenced synthetically by the precipitation space-time distribution, river network structure and basin topography. In general, the research of the runoff space-time distribution theory framework is in the initial phase and has get achievement in simulating and testing precipitation space-time distribution, river network structure and runoff space-time distribution characters. As precipitation is the most important factor influencing runoff space-time distribution, therefore the research of surface water hydrologic process from the point of space-time begins from precipitation, for instance, the former Space-Time Precipitation Process Model[1], and the recent Space-Time Distribution Precipitation Model and Theory[2]. The description of river network structure influences the runoff space-time distribution due to the close relation of river network and basin runoff process. The quantitative description of river network structure originated from Horton (1932) and Strahler (1957)[3,4], and the descriptive method they presented had ever been the basis of river network structure analysis. Nowadays the appearance and development of Digital Elevation Model (DEM) make it possible to abstract high accuracy of river network, and has made progress on the understanding of network self-analogy and dimension analogy[5]. The milestone of researching progress on basin geomorphy runoff response analysis is the proposal of basin instant topography unit graph[6]. Most of the concerning researches appearing lately are based on the river network descriptive method of Horton and Strahler, i.e. from probability deriving the whole runoff process by considering the runoff amount of every river networks to that of high level. This conclusion has supported effectively that the basin instant unit graph (or basin response function) equals basin delay time probability density distribution function. One of the most prominent contributions of the runoff space distribution research on the basis of modern geomorphy data and GIS platform is that the response of basin outlet is the sum of every unit's response function[7], by space resolving basin outlet response function on the basis of unit network linear system. This descriptive method on the basis of unit network makes it possible to determine runoff amount of every point in the basin, and to consider fully the modern digital information in basin response. From this point of view, Olivera and Maidment developed space distribution runoff advance model, however, they referred little to runoff space-time distribution[8]. In addition, the attempt on studying the runoff space-time character and rules by runoff correlative coefficients[9] had not made evident progress due to the difficulty, however the new concern about the problem appears recently[10].

2 The runoff space-time distribution theory

On the basis of the above researches, this paper attempts to present a runoff space-time distribution theory, and adapts it to the runoff correlative coefficient analysis. There are three reasons to do so. First, runoff correlation represents perfectly the basic structure of space runoff, and the theoretical calculating method of runoff correlative coefficient is useful to the runoff interpolation in the regions without data; secondly, because the theoretical analysis of runoff correlative coefficient had ever been the difficult problem in hydrology, solving the problem on theory will play surely an important role in the theory and practice; thirdly, runoff correlative coefficient is easy to prove theory conveniently and straightly.

2.1 Basic hypothesis

The runoff space-time distribution theory the paper presents is based on the following basic hypothesis: (i) The whole basin is segregated to unit network in setting unit dimension, assuming that every unit has only one flowing direction and the elevation of the upper course unit is higher than or equals to that of lower course, so basin river network can be obtained by connecting the units of upper and lower courses, i.e. DEM of single flowing direction; (ii) On river channel, the width of unit network is wide enough to cover main river channel; (iii) For each unit network, the runoff amount of unit equals the net precipitation of the unit. (iv) The evolution of the runoff of every unit i to the next is described by a smoothing (convolution) function h_i:

$$Q_i^{\mathrm{O}}(t)=\int_0^t Q_i^{\mathrm{I}}(\tau)h_i(t-\tau)\mathrm{d}\tau, \tag{1}$$

where Q_i^{I} is the flowing in amount of the unit (including the runoff amount), Q_i^{O} is the runoff amount to the nest unit after evolution (or the contribution of the unit to the flowing-in amount of the next). (v) For the unit with flowing-in amount of the upper course, the runoff amount of the unit equals the sum of all the upper courses flowing-in amount plus runoff amount itself. (vi) When the smoothing function is constant, it in independent on the amount of flowing-in. (vii) Smoothing functions are independent each other.

2.2 Theory Framework

The grid unit assemble in the basin is a space assemble with flowing directions. The basic definition should be made in order to describe confidently this basin grid unit assemble with flowing directions: A_p, the basin assemble or upper course assemble of the grid unit with the outlet point p, i.e. the assemble of the unit grid of all flowing to point p; $O_0^{A_p}$, the outlet assemble of basin assemble A_p, with only one unit grid unit p; $O_1^{A_p}$, 1 unit distance assemble of basin assemble A_p, i.e. the assemble of all the grid units which can reach the basin outlet in case of moving one unit; $O_2^{A_p}$, 2 units distance assemble of basin assemble A_p, i.e. the assemble of all the grid units which can reach the basin outlet in case of moving two units··· $O_n^{A_p}$, n units distance assemble of basin assemble A_p, i.e. the assemble of all the grid units which can reach the basin outlet in case of moving n units. The illustration of the above definitions is shown in fig. 1.

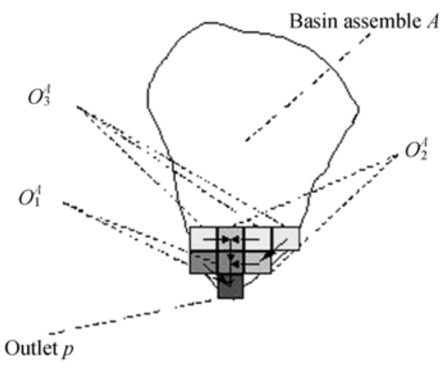

Fig. 1. Illustration of the definition of grid unit assemble.

For any basin assemble A_p, if the longest distance of the units in basin to the basin outlet is n units, then

$$A_p=O_0^{A_p}\cup O_1^{A_p}\cup O_2^{A_p}\cup\cdots\cup O_n^{A_p}. \tag{2}$$

For the purpose of simplification, the superscript of O and p in A_p are deleted, so (2) is simplified to

$$A = O_0 \cup O_1 \cup O_2 \cup \cdots \cup O_n \,. \tag{3}$$

In fact, the linear black box flowing-in evolution is the convolution conversion of a unit from flowing-in to runoff, i.e. the unit graph in hydrology. In physics, the smoothing function represents the parallel translation and of the flowing water, and its shape can reflect the changes of the basin geomorphy. For the purpose of simplification, the area of the unit is assumed to 1 in the following formula (in practical, an area translation coefficient can be multiplied in the formula). If a unit can reach the basin outlet only by flowing n units, its contribution to the basin outlet is nth order convolution conversion of its runoff, written as

$$Q_i(t) = C^n_{\{h_n^i\}}(I_i(t)) \,, \tag{4}$$

where $I_i(t)$ is the runoff amount of unit I, $\{h_n{}^i\}$ is a series smoothing function along lower course of unit i, C represents convolution conversion, and $C^n_{\{h_n^i\}}$ represents nth order convolution conversion of a series functions h_1, h_2, $\cdots$, h_n after being convoluted.

For any basin assemble A_p, if the longest distance of the unit in basin from basin outlet, the basin assemble can be resolved according to (3), so the flowing amount function of basin assemble A can be defined according to the following:

$$fl(A) = Q_A(t) = \sum_{i \in O_n} C^n_{\{h_n{}^i\}}(I_i(t)) + \sum_{i \in O_{n-1}} C^{n-1}_{\{h_{n-1}^i\}}(I_i(t)) + \cdots + \sum_{i \in O_1} C_{h^i}(I_i(t)) + I_{O_0}(t) \,. \tag{5}$$

The right sum in (5) is the independent contribution of every grid unit after the flowing space of the basin outlet is segregated. For every unit, the segregation along the lower course is the high order convolution conversion consisted by a series of smoothing functions.

The important character of smoothing function is established on the research method of detention time which originated Rodriguez-Iturbe and Valdes (1979)[6]. Furthermore, this definition was determined by Gupta et al. (1980)[11]. Through this method, smoothing function corresponds to probability distribution of random variable of detention time in which a unit water quantity is needed to flow from one unit to another unit. This method makes it possible for smoothing function to be expressed by probability distribution of random variable. For nth order convolution conversion consisted by a series smoothing function $\{h_i\}$, random variables X_1, X_2, $\cdots$, X_n represent correspondent probability function h_1, h_2, $\cdots$, h_n. Because h_1, h_2, $\cdots$, h are independent of each other, the detention time of one unit water in the whole course equals the total sum of the detention time of the unit water from one unit to the next. Thus, $X = X_1 + X_2 + \cdots + X_n$ represents the random variable of nth order convolution conversion probability distribution function.

According to the above analysis, high order convolution conversion can be described by a smoothing function which is the convolution of all the smoothing functions along lower course. Assuming the grid units of basin assemble A is NA, then the other form of flowing amount function of the basin assemble A is

$$fl(A) = Q_A(t) = \sum_{i=1}^{N_A} C_{h_i}[I_i(t)], \tag{6}$$

where h_i is the total smoothing coefficient of unit i to the basin outlet.

Compared with that the total runoff amount in the basin is the addition of the distance of units to the outlet in (5), (6) is different. The definition in (5) and the concept of channel travel isochrone or basin width function are corresponding, while (6) is simpler. It can be observed that the flowing process tends to be flat with the increasing basin area.

The flowing quantity function defined in (5) and (6) is a very simple mathematical generalization of basin runoff, because the runoff principle of basin is very complicated. On the form and essence, (6) and Constrained Linear System (CLS) model is unanimous[12]. This paper develops it from unit to unit grid based on DEM, and describes mathematically more comprehensively and completely. Instead of model itself, it emphasizes on the basin runoff space-time distribution theory analysis. They can be the starting point of runoff space-time distribution analysis theory. The development of DEM had made it possible to segregate basin space, so it has provided conditions for the establishment of space-time runoff theory. In addition, (5) and (6) have taken fully into account of the structure of river network, and used the space data of the river network in basin to the extreme limit.

2.3 Correlative coefficient

In practical, the precipitation and runoff of unit is segregative in time. Supposing the duration be Δt, I be a unit in basin assemble A, $Q_i(t) = C_{h_i}(I_i(t))$ represent the contribution of unit i in the basin outlet flow quantity, its segregation format is

$$Q_i(t) = \sum_{k=0}^{\infty} \beta_{ik} I_i(t - k\Delta t), \quad t = 0, \Delta t, 2\Delta t, \cdots \tag{7}$$

where

$$\beta_{ik} = \int_{(k-1)\Delta t}^{k\Delta t} h_i(x)dx, \quad \sum_{k=0}^{\infty} \beta_{ik} = 1.$$

It can be observed that $Q_i(t)$ is the linear filtering output of I_i, β_{ik} ($k = 0, \infty$) is filtering function.

Supposing that I_i is a covariance static series (average and covariance are not changing according to time), its self-covariance function is R_{I_i}:

$$R_{I_i}(v) = Cov\left(I_i(t), I_i(t - v\Delta t)\right) = E\left((I_i(t) - E(I_i))(I_i(t - v\Delta t) - E(I_i))\right).$$

According to linear filtering theory, Q_i is also covariance static series, and

$$E(Q_i) = \sum_{k=0}^{\infty} \beta_{ik} E(I_i) = E(I_i),$$

$$R_{Q_i}(v) = \sum_{k=0}^{\infty} R_\beta(k) R_{I_i}(v-k) \quad \text{where} \quad R_\beta(k) = \sum_{j=0}^{\infty} \beta_{ij}\beta_{i(j+k)}.$$

Supposing that the number of units in basin assemble *A* is NA, then the segregative format of flow quantity function in basin assemble *A* is

$$Q_A(t) = \sum_{i=1}^{N_A} C_{h_i}\left[I_i(t)\right] = \sum_{i=1}^{N_A}\sum_{k=0}^{\infty} \beta_{ik} I_i(t - k\Delta t). \tag{8}$$

The self-covariance matrix of $Q_A(t)$ is

$$R_{Q_A}(v) = Cov\left(Q(t), Q(t - v\Delta t)\right) = E\left((Q(t) - E(Q))(Q(t - v\Delta t) - E(Q))\right)$$

$$= E\left(\left(\sum_{i=1}^{N_A}\sum_{k=0}^{\infty} \beta_{ik}\left(I_i(t - k\Delta t) - E(I_i)\right)\right)\left(\sum_{i=1}^{N_A}\sum_{k=0}^{\infty} \beta_{ik}\left(I_i(t - (k+v)\Delta t) - E(I_i)\right)\right)\right)$$

$$= \sum_{i=1}^{N_A}\sum_{j=1}^{N_A}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty} \beta_{ik}\beta_{jm} Cov\left(I_i(t - k\Delta t), I_j(t - (m+v)\Delta t)\right). \tag{9}$$

For any two arbitrary basin assembles *A* and *B*, the covariance of the two basin outlet flow quantity is

$$Cov\left(Q_A(t), Q_B(t)\right) = E\left((Q_A(t) - E(Q_A))(Q_B(t) - E(Q_B))\right)$$

$$= E\left(\left(\sum_{i=1}^{N_A}\sum_{k=0}^{\infty} \beta_{ik}\left(I_i(t - k\Delta t) - E(I_i)\right)\right)\left(\sum_{j=1}^{N_B}\sum_{m=0}^{\infty} \beta_{jm}\left(I_j(t - m\Delta t) - E(I_j)\right)\right)\right)$$

$$= \sum_{i=1}^{N_A}\sum_{j=1}^{N_B}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty} \beta_{ik}\beta_{jm} Cov\left(I_i(t - k\Delta t), I_j(t - m\Delta t)\right), \tag{10}$$

and its correlative coefficient is

$$Corr\left(Q_A(t), Q_B(t)\right) = \frac{Cov\left(Q_A(t), Q_B(t)\right)}{\sqrt{R_{Q_A}(0) R_{Q_B}(0)}} =$$

$$\frac{\sum_{i=1}^{N_A}\sum_{j=1}^{N_B}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\beta_{ik}\beta_{jm}Cov\left(I_i(t-k\Delta t),I_j(t-m\Delta t)\right)}{\sqrt{\left(\sum_{i=1}^{N_A}\sum_{j=1}^{N_A}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\beta_{ik}\beta_{jm}Cov\left(I_i(t-k\Delta t),I_j(t-m\Delta t)\right)\right)\left(\sum_{i=1}^{N_B}\sum_{j=1}^{N_B}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\beta_{ik}\beta_{jm}Cov\left(I_i(t-k\Delta t),I_j(t-m\Delta t)\right)\right)}}.$$

Supposing that the dispersion of net precipitation of every unit in basin is a constant, then for arbitrary two random variables X and Y, $Cov\left(X,Y\right)=\sqrt{Var(X)Var(Y)}Corr(X,Y)$, put it into the above equation, then

$$Corr\left(Q_A(t),Q_B(t)\right)=$$

$$\frac{\sum_{i=1}^{N_A}\sum_{j=1}^{N_B}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\beta_{ik}\beta_{jm}Corr\left(I_i(t-k\Delta t),I_j(t-m\Delta t)\right)}{\sqrt{\left(\sum_{i=1}^{N_A}\sum_{j=1}^{N_A}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\beta_{ik}\beta_{jm}Corr\left(I_i(t-k\Delta t),I_j(t-m\Delta t)\right)\right)\left(\sum_{i=1}^{N_B}\sum_{j=1}^{N_B}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\beta_{ik}\beta_{jm}Corr\left(I_i(t-k\Delta t),I_j(t-m\Delta t)\right)\right)}}. \quad (11)$$

Eq. (11) is the general form of runoff correlative coefficient of the theory above. It is observed that the runoff correlative coefficient of arbitrary two points can be gained in case if the correlative coefficient of every unit net precipitation and the smoothing coefficient to outlet are known. In practice, more simple formula is used.

2.4 Simplifying formula

In order to simply the formulas above, three handling methods have been used in this research.

(i) Unit impulse shift function is used in smoothing function. Smoothing function is written as $h_i = \delta(t_i)$, and unit impulse shift function $\delta(t_i)$ is defined as $\delta(t)=\begin{Bmatrix}1 & \text{if } t=t_i \\ 0 & \text{otherwise}\end{Bmatrix}$, where t_i is the delay time of flowing water. Unit impulse shift function simulates flowing water shift, so segregative linear filtering coefficient is $\beta_{ik}=\begin{Bmatrix}1 & \text{if } k=t_i/\Delta t \\ 0 & \text{otherwise}\end{Bmatrix}$, so

$$Corr\left(Q_A(t),Q_B(t)\right)$$
$$=\frac{\sum_{i=1}^{N_A}\sum_{j=1}^{N_B} Corr\left(I_i(t-t_i),I_j(t-t_j)\right)}{\sqrt{\left(\sum_{i=1}^{N_A}\sum_{j=1}^{N_A} Corr\left(I_i(t-t_i),I_j(t-t_j)\right)\right)\left(\sum_{i=1}^{N_B}\sum_{j=1}^{N_B} Corr\left(I_i(t-t_i),I_j(t-t_j)\right)\right)}}. \quad (12)$$

(ii) Net precipitation correlative coefficient is independent with runoff delay time. If runoff delay time can be neglected compared with the time step length, for any t_i and t_j,

$Corr\left(I_i(t-t_i), I_j(t-t_j)\right) = Corr\left(I_i(t), I_j(t)\right)$, (12) is changed to be

$$Corr\left(Q_A(t), Q_B(t)\right) = \frac{\sum_{i=1}^{N_A}\sum_{j=1}^{N_B} Corr\left(I_i(t), I_j(t)\right)}{\sqrt{\left(\sum_{i=1}^{N_A}\sum_{j=1}^{N_A} Corr\left(I_i(t), I_j(t)\right)\right)\left(\sum_{i=1}^{N_B}\sum_{j=1}^{N_B} Corr\left(I_i(t), I_j(t)\right)\right)}}. \quad (13)$$

In essence, (13) is unanimous with runoff correlative coefficient formula in Gottschalk (1993)[9]. There are more general significances in (11) than that of Gottschalk, because Gottschalk deduced runoff correlative coefficient formula and applied it in the calculation of yearly runoff correlative coefficient by supposing a rectangle basin. For the long time duration of month and year in middle and small basins, net precipitation correlative coefficient is independent with delay time, i.e. the contributions of all the units to runoff reach to basin outlet in the same period. For a short duration (for example, a day) and great basin, delay time problem and unit smoothing function must be considered.

(iii) The space and time of net precipitation correlative coefficient can be dissolved. That the pace and time of net precipitation correlative coefficient can be dissolved means that net precipitation space-time crossing correlative coefficient can be dissolved into the product of space correlative coefficient and time correlative coefficient:

$$Corr(I_i(k), I_j(m)) = Corr_1(d_{i,j})Corr_2(k-m),$$

for the formula deduced from unit impulse shift function,

$$Corr(I_i(t-t_i), I_j(t-t_j)) = Corr_1(d_{i,j})Corr_2(t_{i,j}),$$

where $d_{i,j}$ is the distance of two units, $t_{i,j} = t_i - t_j$, $Corr_1$ is the space correlative coefficient, and $Corr_2$ is the time correlative coefficient. In other words, the influences of space and time can be dissolved into two functions. For the supposing of unit impulse shift function and constant net precipitation dispersion:

$$Corr\left(Q_A(t), Q_B(t)\right) = \frac{\sum_{i=1}^{N_A}\sum_{j=1}^{N_B} Corr_1(d_{i,j})Corr_2(t_{i,j})}{\sqrt{\left(\sum_{i=1}^{N_A}\sum_{j=1}^{N_A} Corr_1(d_{i,j})Corr_2(t_{i,j})\right)\left(\sum_{i=1}^{N_B}\sum_{j=1}^{N_B} Corr_1(d_{i,j})Crr_2(t_{i,j})\right)}}. \quad (14)$$

3 Practice and verifying of the Wuding River

The Wuding River basin locates in the middle course in the Yellow River, and the river flows across Yikezhao district in the Inner Mongolia Autonomous Region, Yulin and Yan'an in Shaanxi Province. The area of the basin above the Baijiachuan Hydrological Station is 29662 km^2. The data in this analysis include the daily precipitation and daily runoff observation information in 21 a during 1980—2000 in the Wuding River. There are 80 precipitation stations with 21 a data, but there are 2 precipitation stations with incomplete data. So the fundamental data include daily information of 78 precipitation stations and 9 runoff stations (Caoping, Hanjiamao, Dianshi, Mahuyu, Qingyangcha, Hengshan, Lijiahe, Baijiachuan, Dingjiagou), as well as daily observation information with 80—90 a of two runoff stations (Suide and Zhaoshiyao). The Wuding River basin system and precipitation stations are shown in fig. 2.

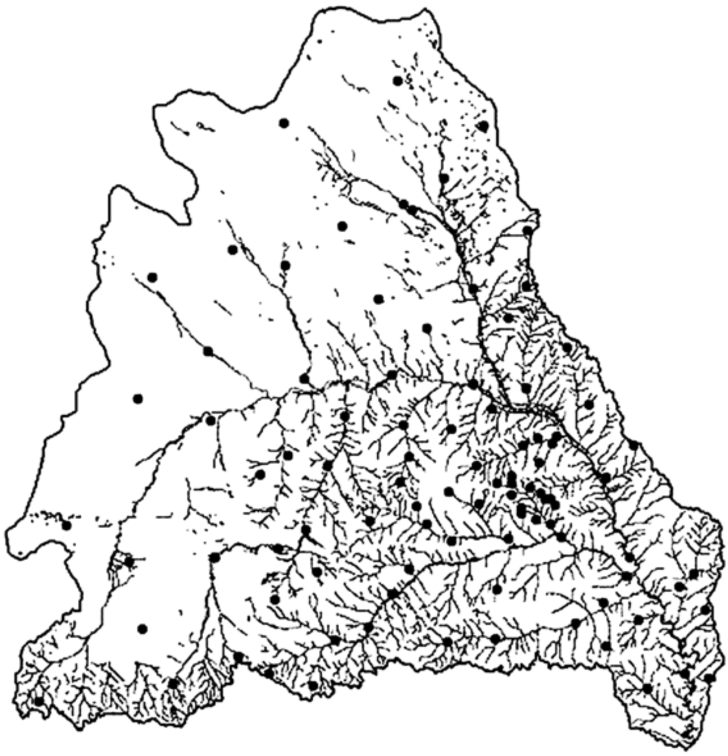

Fig. 2. Graph of the Wuding River basin system and precipitation station distribution.

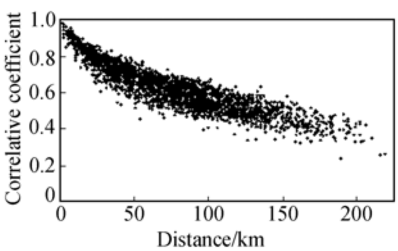

Fig. 3. Relation of the Wuding River basin daily precipitation correlative coefficient and distance.

Generally speaking, the space correlation of precipitation decreases with the increasing of distance. The interrelation between daily precipitation correlative coefficient and space distance of 78 precipitation stations in the Wuding River basin during 1980—2000 is shown in fig. 3. It can be observed that the relation of the space correlation of precipitation and distance is relatively good.

The relation between the correlative coefficient of natural daily runoff (after being restored and modified) of 11 runoff stations in

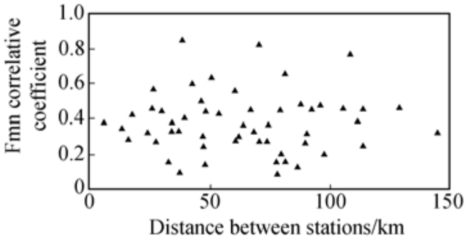

Fig. 4. Relation between daily runoff correlative coefficient in the Wuding River basin and distance.

the Wuding River basin and distance is shown in fig. 4. It can be observed that the relation between runoff correlation and observation station distance is not good.

Because the relation between runoff correlation and distance is not good, the correlative coefficient derived from estimating runoff with distance will cause an high inaccuracy. Eq.(11) is of definitely general significance and has provided a practical method, because it infers synthetically to factors including precipitation, river network structure and basin geomorphy in precipitation runoff course. Up to now, the runoff space-time distribution theory mentioned above had been verified preliminarily in St. Francis basin and Goodwin Creek testing basin in the Mississippi basin in USA. (Wang, 2002)[13]. However, two basins all belong to humid regions where the precipitation-runoff relation is relatively simple, while the Wuding River basin belongs to drought and semi-drought region. Once the theory is verified further, it will be of strongly generality and provide theoretical conduct and implementation method for the future quantization mathematics description of space-time evolution principles of water resources of different basins.

3.1 Analysis of space-time distribution characters in actual measurement precipitation runoff

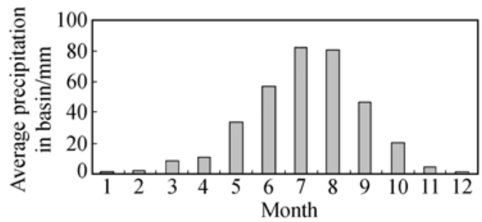

Fig. 5. Monthly average precipitations in 21 years in the Wuding River basin.

Let us first analyze simply the actual measurement precipitation runoff space-time distribution character in the Wuding River basin. Fig. 5 shows the monthly average precipitation of 78 stations in the Wuding River basin during 1980—2000, and the yearly average in 21 a is 350.5 mm. Fig. 6 shows the mean annual runoff above Baijiachuan station after being water use restored, and the average annual runoff in 21 a is 39.1 mm.

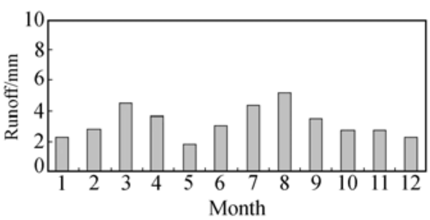

Fig. 6. Average monthly runoff basin above Baijiachuan.

Several conclusions can be gained from figs. 5 and 6: (i) The proportion of runoff in precipitation is relatively small, the basin belongs evidently to drought region; (ii) Runoff monthly change is relatively small, and the runoffs in January, February and December are greater than precipitation. It illustrates that the proportion of underground water in runoff is high; (iii) Runoff increases evidently in February, March (especially in March), and decreases in April and May. It illustrates that the runoff in spring may include melting snow runoff, or it maybe the result of the increasing evaporation in April and May.

Fig. 7 shows the average precipitation in 21 a in 78 precipitation stations within basin and space changing of variation coefficient C_v. It can be observed that the precipitation space homogeneity in statistics is relatively good, and the average tends to increase little from north to south while C_v tends to decrease little from north to south.

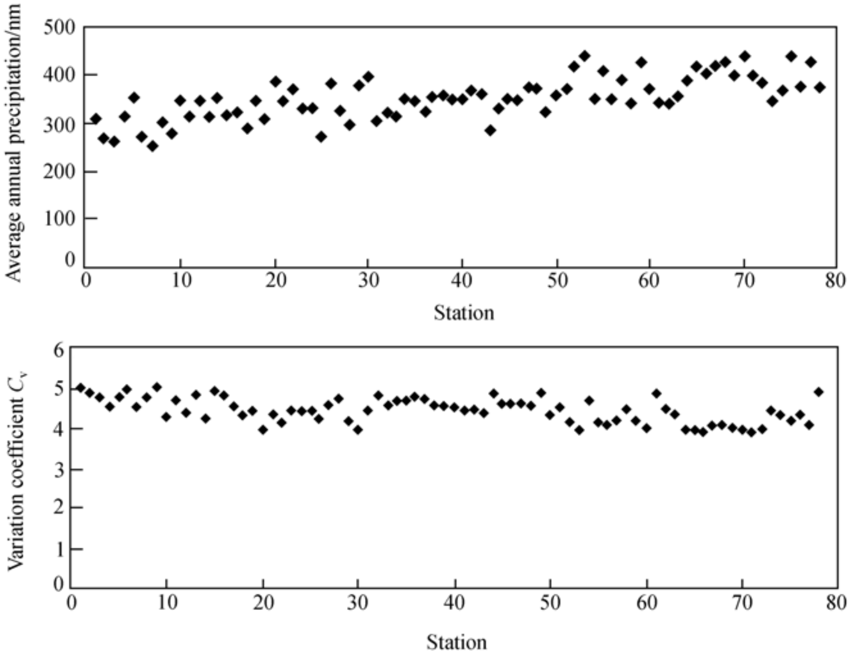

Fig. 7. Average annual actual measurement precipitations in Wuding River basin and variation coefficient C_v space distribution.

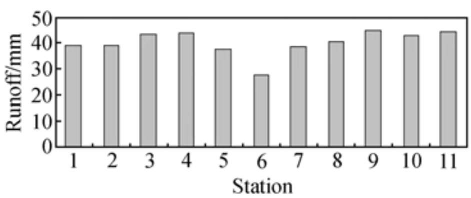

Fig. 8. Average annual actual measurement runoff space distribution of all stations in the Wuding River basin.

Fig. 8 shows the average annual actual measurement precipitation of 11 runoff stations in basin. It can be observed that the actual measurement precipitations of all stations differ little which means that the distribution of runoff in the basin is of equilibrium in space.

Fig. 9 shows the relation between daily runoff variation coefficient and basin catchments area in the Wuding River basin. The nearly declining in straight line can be observed which means that runoff variation decreases in exponent and runoff changes tend to be flat as the basin area increases. It corresponds to the theoretical analysis results above.

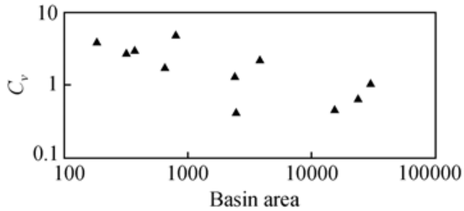

Fig. 9. Relation between daily runoff variation coefficient Cv and basin area double logarithms.

3.2 Verification and analysis of runoff correlation theory

The verification process of this runoff correlation theory is as follows:

First, the Wuding River basin in the Yellow River is clipped from data of GTOPO30 (made by American Geology Plotting Bureau), then the coordinate is altered from geographical coordinate to Albers isohedral coordinate, so a 278×292 grid rectangle DEM covering the Wuding River is made, with the grid size of 830 m. The digital basin water system with resolution 830 m in the Wuding River by pocket storage handling in River Tools, as well as proper modifying of water system made from DEM by actual water system input by the digit equipment, and the fundamental space structure of space-time runoff analysis in the Wuding River basin is established.

Although the theory above is established on the base of basin grid unit, it will take a very long time (about several months) nowadays to calculate in 830 m grid unit calculation, so the unit can be enlarged in practical calculation. This time the Wuding River basin has been divided to units with 100 km^2 on the basis of 830 m resolution digit water system according to the physical basin, and the total number of units is 280. The division of units has been completed automatically by computer, and the information, such as topology relation among all units outlet control stations and river length to basin outlet, has been kept in corresponding files to future use of calculation. Fig. 10 shows the method of unit division, and the unit with 100 km^2 is consist of about 145 grids.

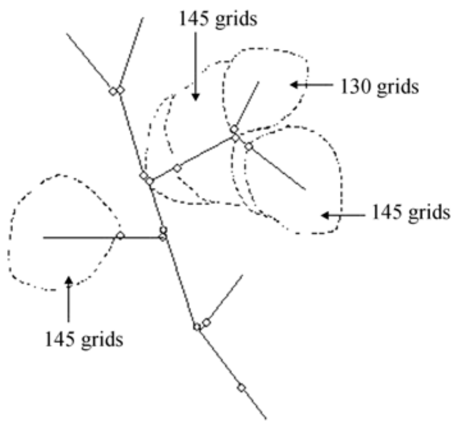

Fig. 10. Illustration of basin unit division method.

Unit division is on the base of control point which is unit outlet. Control point is decided by two criteria: one is about 145 grids of upper course (except other upper course units); the other is that the control point is intersection point of tributaries.

Verification of net precipitation correlative coefficient is the core of deducing runoff correlative coefficient. Some methods are summarized in precipitation runoff actual measurement data in the Wuding River basin for referring in those regions without information. The simplest method is adopting directly precipitation data[9], or synthesizing from precipitation data. It can be observed that the space correlative coefficient of precipitation can be expressed by function of distance. The actual plotting point data, for example, can be summarized by a function $Corr(x) = e^{-\frac{\lambda_1 x}{1+\lambda_2 x}}$, λ_1 = 0.015, λ_2 = 0.013. Correlative coefficient of precipitation time can be synthesized average self-correlative coefficient of single station or multi-stations. Because net precipitation space-time intersection correlative coefficient is needed at last in calculation of runoff correlative coefficient, the best method is to dissolve the space-time correlative coefficient (mentioned above), i.e. space-time correlative coefficient equals the product of time correlative coefficient and space correlative coefficient. If the fitting of this method and actual plotting point data is not good, the method of successive delay time fitting can be used. For daily precipitation in the Wuding River basin, self-correlative coefficient is very small (first order 0.15, second order 0.07), and actual plotting point data shows that from delay 1 precipitation space-time intersection correlative coefficient of actual plotting equals approximately to time correlative coefficient.

Compared with actual plotting data, the inaccuracy of runoff correlative coefficient in the Wuding River calculated from precipitation synthetic space-time correlative coefficient above is very great. It proved that the method summarized directly by precipitation in this basin does not work well. The reason is that the proportion of net precipitation in precipitation is very small because this basin is different from the humid area, so the net precipitation space-time correlative coefficient is far away from precipitation space-time correlative coefficient. The theory itself, of course, needs calculation by net precipitation.

There are many methods in calculate net precipitation (or runoff). For runoff correlation analysis, it is recommended that simplest method be used, such as infiltration index method, i.e. ϕ-index (the part of precipitation more than ϕ is net precipitation). The reason is that the net precipitation space-time correlative coefficient is easy to be summarized to do so. The ϕ-index of constant space is used because the precipitation runoff space distribution in the Wuding River basin is very even. ϕ = 25 mm after the trial calculation. Fig. 11 shows the result of net precipitation correlative coefficient fitting by the same method of fitting point fig. 3 above, and it also shows the relation between net precipitation space correlative coefficient and distance in 78 stations, in which the point data is the average of all actual measurement correlative coefficients within the distance with every 1 km. The fitting curve of space net precipitation correlative coefficient is $Corr(x) = e^{-\frac{\lambda_1 x}{1+\lambda_2 x}}$. After analyzing the net precipitation time and space-time intersection correlative coefficient, it has been observed that they can be neglected because they are very small from delay 1. So the synthesized net precipitation space-time correlative coefficient in the Wuding River basin is

$$Corr\left(I_i(t-k\Delta t), I_j(t-m\Delta t)\right)=\begin{cases} e^{-\left(0.03\,d_{i,j}/(1+0.011\,d_{i,j})\right)} & \text{if} \quad k=m \\ 0 & k\neq m \end{cases},$$

where $d_{i,j}$ is the distance between two units.

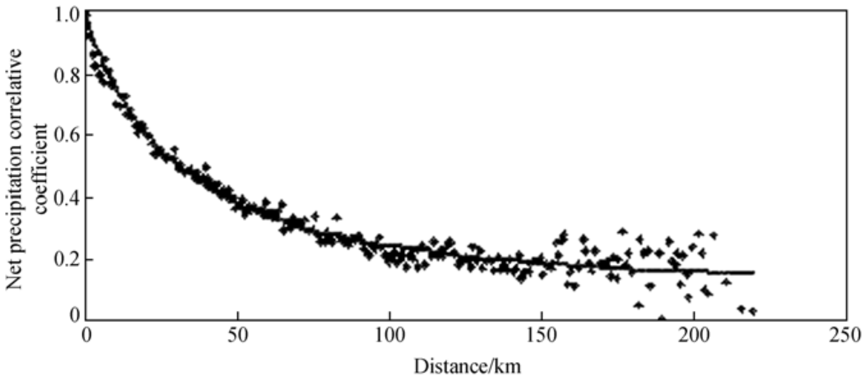

Fig. 11. Relation between net precipitation correlative coefficient and distance.

The next step is the verification of smoothing function of every unit. According to the experience in two humid basins in USA (Wang, 2002)[13], a satisfactory precipitation correlative coefficient calculating result can be obtained by unit impulse shift function and simplified formula above, (12)—(14). The calculating result in the Wuding River basin, however, shows that this kind of generalization is not suitable for this basin, and the reason may be that the basin locates in the drought region and the basin area is very large. This paper attempts the following method.

Concerned that the area of every unit is about 100 km^2, within the basin an observed precipitation station, whose basin area is close to the unit area, is chosen. The synthetic analysis in undertaken directly from the precipitation station runoff observed, and an experienced unit graph is considered as smoothing function of every unit (attention: instead of smoothing or runoff function of flowing unit, this is responding function of unit itself in concept, and the handling of this paper is a kind of approximate method). The total smoothing function of every unit to outlet is the convolution of successive unit smoothing function. According to the idea above, Caoping station (whose basin runoff area is 187 km^2) has been chosen as a typical runoff station. The basin daily runoff zero dimension unit graph has been synthesized from many independent precipitation runoff procedures, and this unit graph is regarded as the smoothing function of every unit in the basin (shown in table 1).

Table 1 Smoothing function of every unit

Delay time/d	0	1	2	3	4	5	6	7	8	9	10
Function	0.4776	0.1625	0.0815	0.0797	0.0574	0.0355	0.0341	0.0139	0.0138	0.0136	0.0135

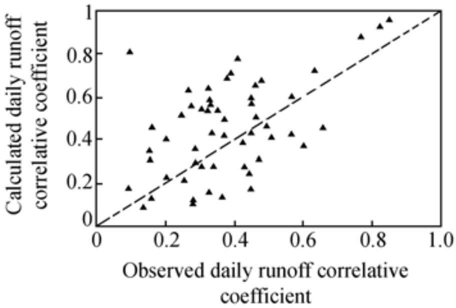

Fig. 12. Comparisons between daily runoff correlative coefficient calculated in the Wuding River basin and actual plotting value.

Up to now, the runoff correlative coefficient of arbitrary two units outlet in the Wuding River can be calculated by formula (11), according to the handled basin unit river network topology relation, space-time net precipitation correlative coefficient and smoothing function of every unit. Fig. 12 shows the comparisons of the correlative coefficients between calculating results of 11 observed stations and calculation of observed data. The average absolute error sum of squares of daily runoff correlative coefficient of observed and calculating is 0.0423.

It can be observed that the result is not perfect. Compared with scattered point data in fig. 4, however, the evident linear tend is shown in fig. 12. This proves that the theory in this paper can provide an effective conduct to the estimation of runoff correlative coefficient in the Wuding River basin, so preliminary verification of the runoff space-time distribution theory has been obtained in the runoff correlation analysis. In fig. 12, there is point data with a large error in correlative coefficient in Suide and Lijiahe stations. Because Lijiahe lies on the upper course to Suide, the correlative coefficient (calculating value 0.81) should be very large in theory, however, the correlative coefficient of actual daily runoff series is only 0.09. The reason may be the influence of other human behaviors except water use, and further analysis will be undertaken in future water resources research. In addition, there is a tendency of daily runoff correlative coefficient to be large in calculation, and the reason may be that besides the theory itself, the influence of human behavior, except water use, causes the actual runoff correlative coefficient systematically to tend to be small. Water use has been restored in this paper, but the factors of other human behavior influence, such as reservoir and silt mud dam, have not due to the limitation of information. This also may cause the calculated daily runoff correlative coefficient to depart that of actual plotting series restored by water use.

4 Conclusion

This paper has proposed the runoff space-time distribution theory and established scientific fundament for water resources space-time evolution rules research. It has been verified preliminarily in the Wuding River basin in the Yellow River basin, and shown good theoretical advance and practical adoptability. It must be mentioned, however, that the theory is needed to be verified further through runoff stimulating due to, restriction of information and many generalizations in runoff correlation analysis because the theory mentioned in this paper is in the practicing procedure. In addition, nowadays the object described by this theory focuses on restored natural runoff space-time character, and the human behavior influence needed to be further considered in future, in order to make it to play more important roles in more range of water resources practice.

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