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Complexity analysis of precipitation in changing environment in Chien River Basin, China

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Abstract: The hydrological processes influenced by the multiple factors of climate, geography, vegetation, and human activities are becoming more and more complex, which is an important characteristic of hydrological systems. The different complexity distributions of precipitation processes of the Chien River Basin (a sub-basin of the Minjiang Basin) in two periods (from 1952 to 1980, and from 1981 to 2009) are illustrated using the fractal based on the continuous wavelet transform (CWT). The results show that (1) at the basin scale the precipitation process in the latter period is more complex than in the former period; (2) the maximum value of the complexity distribution moved from the east to the middle; and (3) through analysis of the time-information and space-information concealed in this complexity change, the precipitation characteristics in the changing environment in the basin can be illuminated. This study could provide a reference for research on disaster pre-warning in changing environments and for integrated water resources management in the local basin.

Key words: *characteristic analysis; precipitation complexity; continuous wavelet transform; fractal; Chien River Basin*

1 Introduction

Research on complexity has attracted increasing attention since the 1970s, as some properties of systems cannot be interpreted by traditional methods and indices (Lempel and Ziv 1976). Mandelbrot (1983) put forward the concept of the fractal and showed that the phenomenon of complexity in nature has self-similarity that can be interpreted by fractal. In recent decades, the study of hydrologic complexity has become a hot issue in hydrological science. Li et al. (1999) calculated the wavelet estimation values of annual flow peak series using the Hust index based on the continuous wavelet transform (CWT) and compared this method with other methods. Li and Wang (2002) calculated the information fractals of the

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drought and waterlogging series of the Huaihe River and Yangtze River in Anhui Province, China, using Shannon entropy, and analyzed the complexity characteristics of these drought and waterlogging series based on the results. Wang et al. (2004) diagnosed features such as break and jump (also features of complexity) of hydrological time series applying the box fractal based on CWT. Wang et al. (2005) analyzed the complexity characteristics of the runoff at typical stations of the Yellow River using the fractal and made comparison with the R/S method. Luan et al. (2010b) analyzed the complexity distributions of precipitation processes in a semi-arid region using a fractal based on CWT in different periods and discussed the reasons for the analysis results. But the study on complexity distributions in humid regions is limited. In this study, the law of complexity distributions in a typical humid region in changing environment was studied using the fractal and the explanation for the related results is also given.

2 Method

2.1 Continuous wavelet transform (CWT)

Wavelet transform (WT) evolved from the Fourier transform (FT) and short-time Fourier transform (SFT) in the 1980s (Morlet et al. 1982). In contrast to other transform methods, wavelet analysis has the multi-resolution analysis function and auto-adjustable time-frequency window, which can obtain more information from series (Morlet et al. 1982). When the wavelet function $\psi\left(t\right)$ is consistent and steady, the principle of CWT is as follows (Sidney et al. 1997; Luan et al. 2010c):

$$W_X\left(a,b\right)=\left|a\right|^{-\frac{1}{2}}\int X(t)\psi^*\left(\frac{t-b}{a}\right)\mathrm{d}t \tag{1}$$

where $W_X\left(a,b\right)$ is the coefficient of WT, a is the scale parameter (expansion parameter), $\left|a\right|^{-\frac{1}{2}}$ is the standardization factor, b is the time position parameter (translation parameter), t is time, $\psi^*\left(t\right)$ is the complex conjugate of $\psi\left(t\right)$, and $X\left(t\right)$ is a hydrological series here.

The WT coefficient $W_X\left(a_j,b\right)$ of the hydrological series in multi-scale $a_j\left(j=1,2,\cdots,M\right)$ (M is the number of scale) can be obtained by Eq. (1), where $b=1,2,\cdots,N$ (N is the length of the hydrological series).

2.2 Exponent of energy spectrum

The precipitation series is a typical self-similarity series and its WT coefficients in different scales, $W_X\left(a_j,b\right)$, can be characterized by their self-similarity (Barenblatt 1996). According to the fractal theory (Wornell 1995), the exponent of energy spectrum (α^*) is a feasible index reflecting the self-similarity of the series; the analytic relations between the energy spectrum $\Gamma_X\left(\omega\right)$, α^*, and the frequency ω can be shown as follows:

$$\varGamma_X(\omega) \propto \frac{1}{\omega^{\alpha^*}} \tag{2}$$

where $-1 < \alpha^* < 3$. For $\omega = 1/a$, the analytic relations between the energy spectrum $\varGamma_X(a)$, α^*, and a can be shown as follows:

$$\varGamma_X(a) \propto a^{\alpha^*} \tag{3}$$

The energy spectrum in multi-scale a_j (Wornell 1995; Li et al. 1999), $\varGamma_X(a_j)$, can be computed as follows:

$$\varGamma_X(a_j) = \sum_{b=1}^{N} \frac{\left[W_X(a_j, b)\right]^2}{N} \qquad (j = 1, 2, \cdots, M) \tag{4}$$

When each side of the function in Eq. (3) is computed in logarithm (the base is 2), the regression equation can be obtained through a combination with Eq. (4):

$$\log_2\left[\varGamma_X(a_j)\right] = \log_2\left\{\sum_{b=1}^{N} \frac{\left[W_X(a_j, b)\right]^2}{N}\right\} = \alpha^* \log_2 a_j + c_0 \tag{5}$$

where c_0 is a constant, and α^* is equal to the slope of the fitted curve (Wang et al. 2004; Wang et al. 2005).

2.3 Significance test of exponent of energy spectrum

Since the exponent of the energy spectrum is obtained through curve fitting, it is essential to use the significance test to verify its stability. R_α^2 is the critical value of the correlation coefficient between $\log_2\left[\varGamma_X(a_j)\right]$ and $\log_2 a_j$ at a given significance level α, and can be expressed as

$$R_\alpha^2 = \frac{F_\alpha(1, n-2)}{(n-2) + F_\alpha(1, n-2)} \tag{6}$$

where $F_\alpha(1, n-2)$ is the value of *F* distribution at the significance level α, and *n* is the total number of spots in the fitting curve (Freedman et al. 2007) and is equal to the scale number of a_j, which means *n* = *M*. R^2 is the correlation coefficient between $\log_2\left[\varGamma_X(a_j)\right]$ and $\log_2 a_j$. If $R^2 > R_\alpha^2$, the regression equation and the exponent of the energy spectrum are both acceptable at a given confidence level ($1-\alpha$).

2.4 Fractal dimension based on CWT

If α^* is acceptable, the fractal dimension (*D*) can be computed as follows (Falconer 2003):

$$D = \begin{cases} 1.5 - \dfrac{\alpha^*}{2} & (-1 < \alpha^* < 1) \\ 2.5 - \dfrac{\alpha^*}{2} & (1 < \alpha^* < 3) \end{cases} \tag{7}$$

That is to say, the fractal of the series based on CWT can be calculated according to Eqs. (1) through (7).

A hydrological series is an observed sequence, which has a variety of changes and uncertain characteristics. WT has the function of multi-resolution analysis, so the characteristics at multiple scales of hydrological series can be determined by WT and the complexity of the hydrological series can be interpreted more precisely using the fractal based on CWT. Wang et al. (2005) proved that it was feasible to use the fractal based on CWT as the index for analyzing the complexity of hydrological series.

3 Application

3.1 Study region and data selection

The Chien River Basin, with an area of 16.4×10^3 km^2, was chosen in this study. The Chien River is a large tributary of the Minjiang River and is located in the northwest of Fujian Province in China. It lies in the subtropical and ocean monsoon climate zone, so the runoff is abundant. The mean annual discharge is 16.7×10^9 m^3 and the mean annual area precipitation is 1 675 mm, of which more than 80% is concentrated in the flood season (from May to September). Two groups of annual precipitation series of nine gauging stations located on the main branches of the Chien River Basin were applied in this study. One group consists of the series from 1952 to 1980 and the other consists of the series from 1981 to 2009. A length of the hydrological series of 29 and a scale number of 16 were chosen for this study. The locations and the mean annual precipitation of the selected stations in this basin are listed in Table 1.

Table 1 Locations and mean annual precipitation of gauging stations

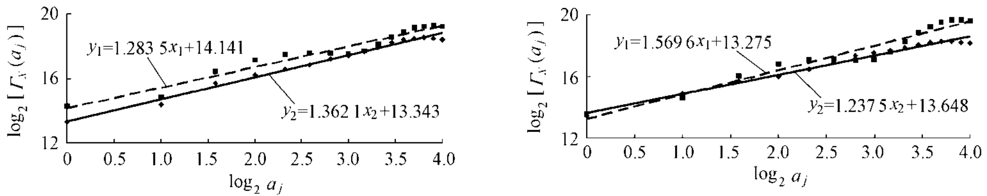
Gauging station	Latitude	Longitude	Mean annual precipitation (mm)	
			1952-1980	1981-2009
Pucheng	27°55′N	118°32′E	1 728	1 687
Shuiji	27°24′N	118°20′E	1 708	1 609
Wuyishan	27°46′N	118°02′E	1 919	1 868
Songxi	27°31′N	118°48′E	1 656	1 604
Dongyou	27°10′N	118°38′E	1 598	1 547
Zhenghe	27°22′N	118°51′E	1 633	1 585
Qilijie	27°01′N	118°17′E	1 642	1 583
Jianyang	27°20′N	118°07′E	1 734	1 584
Masha	27°23′N	117°50′E	1 776	1 723

3.2 Wavelet function selection

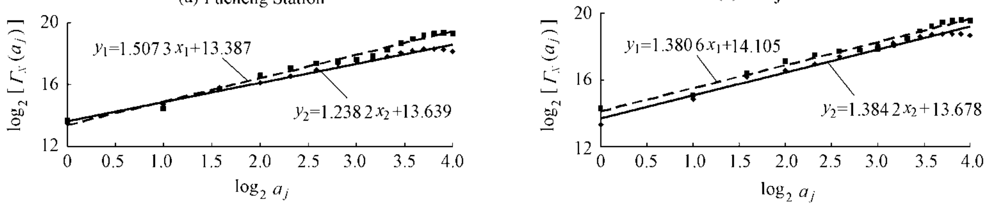
Wavelet functions have various forms. The way of selecting a fit function is critical to the application of the wavelet function. The Daubechies wavelet is a compactly supported function with biorthogonal characteristics; its wavelet coefficient can show more features of the hydrological series (Daubechies 1988). Daubechies 7 (abbreviated for Db7) was selected for this study.

3.3 Application and significance test

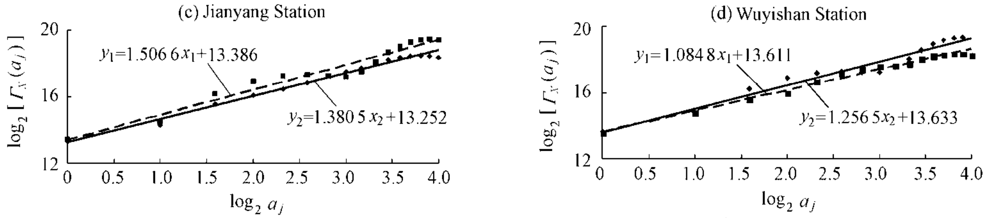
With the method used in this study, the regression equations (shown in Fig. 1), R^2 of each regression equation, and the corresponding exponent of the energy spectrum can be obtained (Table 2). It is shown in Table 2 that R^2 in each regression equation is larger than 0.90, while the critical value of R_α^2 at a 1% significance level is equal to 0.3876. Thus, all the regression equations pass the significance test and these results are acceptable to be used for computing the fractal based on CWT according to the theory of statistics (Freedman et al. 2007); the complexity analysis of these precipitation series using the indices above is feasible.



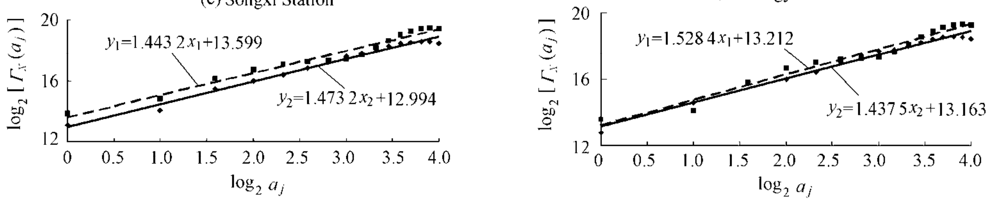
(a) Pucheng Station (b) Shuiji Station



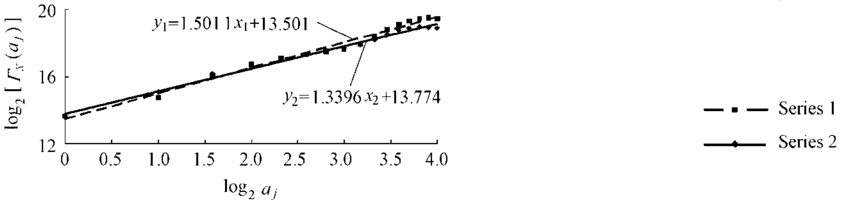
(c) Jianyang Station (d) Wuyishan Station



(e) Songxi Station (f) Dongyou Station



(g) Zhenghe Station (h) Qilijie Station



(i) Masha Station

Fig. 1 Regression equations of energy spectrum of two precipitation series at nine gauging stations

Table 2 Fractal dimensions, exponents of energy spectrum, and correlation coefficients of regression equations during two periods in Chien River Basin

Gauging station	Period from 1952 to 1980			Period from 1981 to 2009		
	α^*	D	R^2	α^*	D	R^2
Pucheng	1.284	1.858	0.956 3	1.362	1.819	0.987 5
Shuiji	1.570	1.715	0.948 4	1.238	1.881	0.984 0
Wuyishan	1.381	1.810	0.977 7	1.384	1.808	0.974 8
Songxi	1.507	1.747	0.952 7	1.381	1.810	0.986 5
Dongyou	1.085	1.958	0.953 8	1.257	1.872	0.983 8
Zhenghe	1.443	1.778	0.968 3	1.473	1.763	0.986 3
Qilijie	1.528	1.736	0.968 3	1.438	1.781	0.981 8
Jianyang	1.507	1.746	0.983 0	1.238	1.881	0.984 9
Masha	1.501	1.749	0.983 9	1.340	1.830	0.991 7
Mean of D		1.789			1.827	

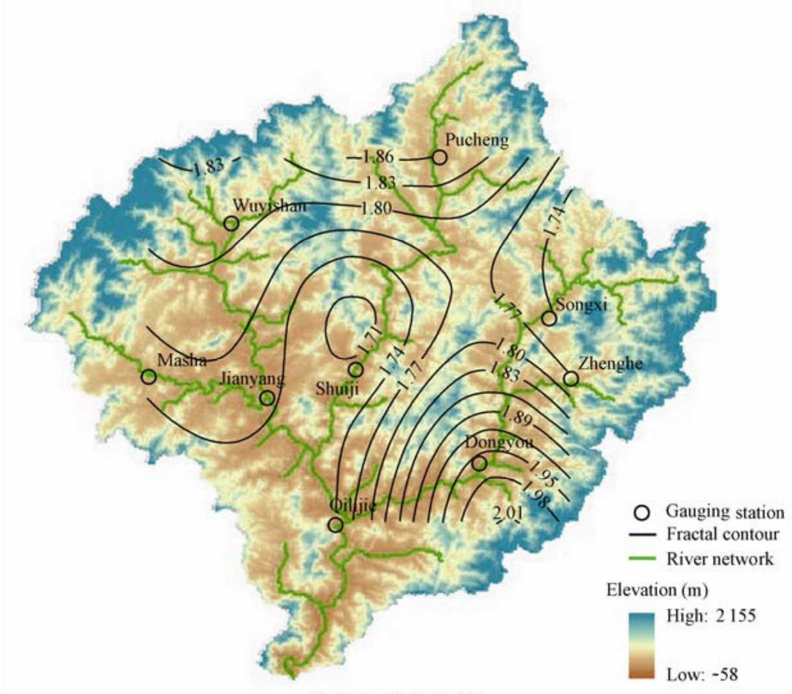
4 Results and analysis

The fractals based on CWT of precipitation series at different gauging stations of the Chien River Basin in two different periods can be calculated according to Eq. (7). These fractals indicate the precipitation complexity of each station, which are listed in Table 2.

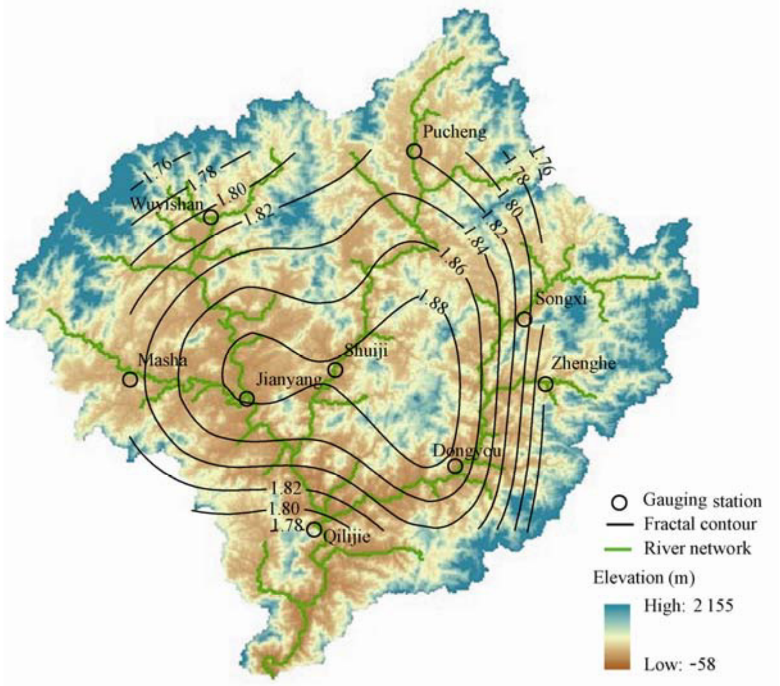
Thus, the different complexity distributions of precipitation in these two periods in the Chien River Basin can be sketched out based on the fractals and the locations of each station (Table 1 and Table 2), which are displayed in Fig. 2(a) and Fig. 2(b), respectively.

The complexity of the precipitation series is the reflection of a variety of precipitation characteristics and factors. According to the actual factors of the study basin, the differences of precipitation complexity in different periods can be analyzed and explained.

It can be seen from Table 2 that the mean annual precipitation fractal was 1.789 in the former period and 1.827 in the latter period, which means that the mean annual precipitation complexity in the latter period was larger than in the former period. In contrast, the mean annual precipitation at the nine stations all decreased in the latter period, as shown in Table 1. The increase of precipitation complexity shows that in the study basin the mean annual distribution of rainfall was more disorderly in the latter period. That is to say, the frequency of random precipitation increased in the latter period, meaning that at the scale of the whole basin, the mean frequency of heavy rainfall or light rainfall increased, but that of the regular seasonal rainfall decreased. This is the reason for the increase of complexity and the decrease of the total rainfall. This result shows that the complexity change of precipitation is not correlative with the total rainfall but with the frequency of rainfall. Furthermore, precipitation is one of the most sensitive climate factors and its annual frequency variance is the reflection of the climate change. The analysis of the precipitation complexity change shows that extreme meteorological events (especially flooding and water logging) increased in the study basin, which is consistent with the authoritative research (EBNARCC 2007). Therefore, it can be concluded that, in the study basin, climate change is an important factor of the precipitation complexity change.



(a) From 1952 to 1980



(b) From 1981 to 2009

Fig. 2 Complexity distributions of precipitation using fractal in Chien River Basin during two periods

Table 2 and Fig. 2 show that in the Chien River Basin there was little precipitation complexity change in the northwestern region (Wuyishan); the precipitation complexity decreased in the eastern regions (Dongyou and Zhenghe) and increased in the middle and southwestern regions (Qilijie, Shuiji, Jianyang, and Masha). Thus, the maximum value of the precipitation complexity contours moved from the southeast to the middle of the basin, which demonstrates that the mal-distribution of the annual precipitation series increased in the central region and decreased in the southeastern region. According to the fractal theory and the analysis of the relationship between the fractal of the precipitation series and the climate change, this difference means that the frequency of the meteorological phenomena increased in the central region of the study basin but decreased in the southeastern region. The topographic element, another factor, should be taken into account for illustration. The vapor for some stations (such as Dongyou and Zhenghe) located on the leeward slope of mountains is absent and the probability of short-term random precipitation is little. Therefore, the corresponding precipitation complexity decreased. It is also shown that both the landforms and the climate (the main factor) influence the frequency and other characteristics of rainfall, and, thus, the precipitation complexity is also influenced.

It can be seen from Fig. 2, from the trough value of the complexity contours before 1980 to the peak value after 1980, that the precipitation complexity at the stations (Jianyang and Shuiji) in Jianyang City increased remarkably after 1980. Human activities, another factor, can be taken into account to explain this phenomenon. Jianyang City is the industrial base for the north of Fujian Province, and its gross domestic product (GDP) has ranked among the top ten cities in Fujian Province in recent decades. Urbanization, industrialization, and other human activities generate much suspended particles and heat energy, which changes the formation conditions of precipitation and the precipitation properties both in the time-frequency domain and in the space domain, and thus causes the precipitation complexity change in the course of time. Therefore, the increase of precipitation complexity at Jianyang and Shuiji stations results from both climate change and various human activities.

5 Conclusions

The Chinese environment has changed sharply in recent decades due to climate change and various human activities, which causes changes in precipitation characteristics in both the time domain and frequency domain, and thus causes the precipitation complexity to increase. This means that the complexity change of precipitation processes in different regions is a reflection of the changing environment.

In this study, several types of rainfall properties in the changing environment were examined through analysis of the complexity change of precipitation processes at different stations during different periods using a fractal based on CWT in the Chien River Basin. From the study results and similar research results from typical semi-arid regions (Luan et al. 2010a, 2010b), the following conclusions can be drawn:

(1) The complexity of the annual precipitation process is not correlative with the total rainfall but with the frequency of rainfall. In different typical climate zones, the change of rainfall frequency is different and the precipitation characteristics and the corresponding changes of precipitation complexity with climate change are also different: in the Chien River Basin (a typical humid region), the random rainfall frequency and the precipitation complexity increased but the total rainfall amount decreased after 1980, but in the typical semi-arid region (Luan et al. 2010a, 2010b), the complexity (frequency) and the total rainfall both decreased.

(2) The complexity changes of precipitation processes at different stations during the two periods are different and these changes show different characteristics of the influences of the topographic factors and climate factors. After 1980, the frequency of random rainfall and the precipitation complexity deceased only in the leeward of the eastern region in the Chien River Basin, but increased throughout the whole mountain area in the typical semi-arid region (Luan et al. 2010a, 2010b).

(3) Various human activities such as urbanization, industrialization, and population growth increase the precipitation complexity. These activities are remarkable both in the typical humid region and in the typical semi-arid region. Correcting gauging errors caused by human activities is a problem in hydrology, which needs to be solved in the future research.

(4) The study results conform with the *National Assessment Report of Climate Change* (EBNARCC 2007) and other authoritative research, proving that the method used in the present study is feasible and can be adopted to study precipitation properties in the changing environment. Further research on other typical climate zones will be done in the future.

References

Barenblatt, G. I. 1996. *Scaling, Self-similarity, and Intermediate Asymptotics: Dimensional Analysis and Intermediate Asymptotic*. New York: Cambridge University Press.

Daubechies, I. 1988. Orthonormal bases of compactly supported wavelets. *Communications on Pure and Applied Mathematics*, 41(7), 909-996. [doi:10.1002/cpa.3160410705]

Editoral Board of National Assessment Report of Climate Change (EBNARCC). 2007. *National Assessment Report of Climate Change*. Beijing: Science Press. (in Chinese)

Falconer, K. 2003. *Fractal Geometry: Mathematical Foundations and Applications*. 2nd ed. West Sussex: John Wiley & Sons.

Freedman, D., Pisani, R., and Purves, R. 2007. *Statistics*. 4th ed. New York: W. W. Norton & Co.

Lempel, A., and Ziv, J. 1976. On the complexity of finite sequences. *IEEE Trans on Information Theory*, 22(1), 75-81.

Li, D., and Wang, F. 2002. A study on information quantity and fractal of disaster sequences of regional drought and waterlogging. *Journal of Catastrophology*, 17(2), 11-16. (in Chinese)

Li, X. B., Ding. J., and Li, H. Q. 1999. The wavelet estimation of Hurst coefficient in hydrological time series. *Journal of Hydraulic Engineering*, 30(8), 21-25. (in Chinese)

Luan, Q. H., Chen, L. X., and Cheng, Y. 2010a. Analysis and comparing of the distribution of precipitation complexity in two typical regions in changing environment, China. *Proceedings of 2010 International Workshop on Chaos-Fractal Theories and Application*, 391-394. Washington: IEEE Computer Society. [doi:10.11098 /IWCFTA.2010.40]

Luan, Q. H., Qin, D. Y., Yuan, F., He, J., and Wu, T. B. 2010b. Analysis of the distribution of precipitation complexity under climate change in Handan, China. *Proceedings of the 9th International Conference on Hydroinformatics*, 702-709. Beijing: Chemical Industry Press.

Luan, Q. H., Yuan, J., Ma, Z. Z., Hao, X. B., and Wu, T. B. 2010c. Periodicity and trend analysis of precipitation in multi-time scale in plain regions of Handan, China. *Applied Mechanics and Materials*, 29-32, 2739-2744. [doi:10.4028/www.scientific.net/AMM.29-32.2739]

Mandelbrot, B. B. 1983. *The Fractal Geometry of Nature*. 3rd ed. New York: W. H. Freeman and Company.

Morlet, J., Arens, G., Fourgeau, E., and Giard, D. 1982. Wave propagation and sampling theory and complex waves. *Geophysics*, 47(2), 222-236. [doi:10.1190/1.1441329]

Sidney, B. C., Gopinath, R. A., and Guo, H. T. 1997. *Introduction to Wavelets and Wavelet Transforms: A Primer*. Bergen County: Prentice Hall.

Wang, W. S., Zhao. T. X., and Ding, J. 2004. Study on change characteristics of hydrological time series with continuous wavelet transform. *Journal of Sichuan University (Engineering Science Edition)*, 36(4), 6-9. (in Chinese)

Wang, W. S., Xiang, H. J., Huang W., J., and Ding, J. 2005. Study on fractal dimension of runoff sequence based on successive wavelet transform. *Journal of Hydraulic Engineering*, 36(5), 598-601. (in Chinese)

Wornell, G. 1995. *Signal Processing with Fractal: A Wavelet Based Approach*. Bergen County: Prentice Hall.