Study on Soil Stable Infiltration Characteristics at Different Depths in Dongliao River Basin, Northeast China

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Abstract—**Based on infiltration experiments in the central Dongliao River Basin, the multifractal analysis method was used to investigate the spatial variability of soil stable infiltration characteristics (SSIC) at different depths and the correlation to soil physical properties. Then the conversion models of soil stable infiltration characteristics at different depths were established. The results showed that soil stable infiltration characteristics at different depths were multifractal. The spatial variabilities of soil stable infiltration rates at the depths of 30cm and 50cm were mainly caused by those of clay content, organic matter content and soil moisture content at the depth of 10cm at point scale. The multi-scale analysis results showed that the soil stable infiltration rates at the depths of 30cm and 50cm had the most significant correlation to soil moisture content at the depth of 10cm, soil bulk density, organic matter content and total N content. Two conversion models of soil stable infiltration characteristics at the depths of 30cm and 50cm were established with smaller computational errors, based on multifractal theory. The correlation coefficients of two models were 0.996 and 0.958 respectively, corresponding to RMSE of 0.048 and 0.050.**

Keywords- soil stable infiltration; multifractal analysis; soil physical property

I. INTRODUCTION

The study on soil stable infiltration characteristics at different depths is of great significance to soil moisture conservation, water runoff calculation, efficient use of water resources and many other aspects [1]. However, apart from the influences of soil structure [2], soil texture, land use patterns and other factors, soil infiltration characteristics also show significant spatial variability, leading to many difficulties in the research on this field [3]. Since the 20th century, a lot of studies have been conducted on this field, but mostly on a small scale [4]. In order to acquire larger-scale soil infiltration parameters, many scholars select various statistical methods to study the spatial variability of soil infiltration characteristics, based on massive field experiments. In addition, spatial variability of soil infiltration characteristics is the result of joint action by influencing factors at multiple scales, and the influence intensities of various factors change with scales. Therefore, fully considering the influencing characteristics and intensity of soil physical properties on soil infiltration characteristics at different scales, the function relation between soil infiltration characteristics and measurable soil physical properties is established, i.e., conversion model of soil characteristics, which is an important solution to obtain larger-scale soil infiltration parameters.

Multifractal method is an effective tool to analyze the spatial variability of study object at multi-scale and its correlation to influencing factors, which has been applied to many fields [5-14]. However, aiming at soil infiltration characteristics under natural conditions, especially for soil stable infiltration characteristics at different depths, rare studies exist on the multi-scale spatial variability and conversion model of soil characteristics. Accordingly, based on multifractal method, an analysis was conducted on multi-scale spatial variabilities and correlations of soil physical properties and soil stable infiltration characteristics at the depths of 30cm and 50cm in this study, upon which, the conversion model of soil characteristics between soil stable infiltration characteristics and soil physical properties was established.

II. MATERIALS AND METHODS

A. Study Area

Dongliao River is the tributary on the left side of the upper reaches of Liaohe River, originated from Anle Village, Yanping Town, Dongliao County, crossing Liaoyuan, Yitong, Lishu, Huaide and Shuangliao in Jilin Province, Xifeng, Changtu, Kangping and other cities and counties in Liaoning Province (see Fig.1). Dongliao River Basin's length and area is 44.8×10^4 m and 113.06×10^8 m^2 respectively[15]. It is roughly divided into three segments, of which, the upper reaches are the segment above Erlongshan Reservoir as a low-mountain hilly area with general altitude of 200-500m, where soil types primarily consist of dark brown soil and planosoil; the middle reaches are the segment from Erlongshan Reservoir downwards to Chengzishang Station as a hilly area with bended rivers and good vegetation, where soil types primarily consist of black soil and meadow soil; the lower reaches are the segment from Chengzishang Station downwards to Sanjiangkou Iron Bridge in Siping-Qiqihar Railway Line, where soil types primarily consist of meadow soil, salinized chernozem and steppe aeolian sandy soil.

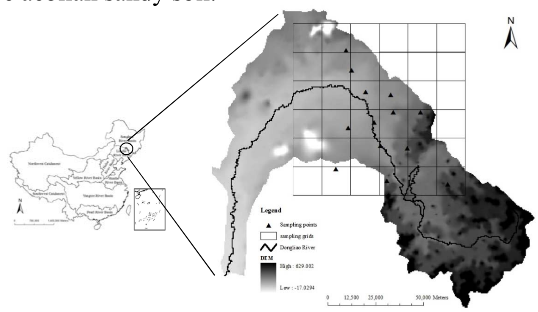

Figure 1. Distribution of Dongliao River Basin and sampling points.

Dongliao River Basin is controlled by the Pacific low and Siberian high with obvious four seasons. The precipitation is decreasing from upper to lower reaches, and multi-year average precipitation is reduced from top 700mm to bottom 450mm. The precipitation is distributed unevenly within the year, of which, that from June to September accounts for 75% of annual precipitation, and that of July and August accounts for 50%. Inter-annual precipitation change is also decreasing from west to east. The precipitations in rainy and dry years are different from measured maximum and minimum annual precipitation by 2 to 4 times. Surface runoff is distributed similarly to annual precipitation. From the low-mountain hilly areas in the upper reaches to the plains in the lower reaches, the runoff depth is reduced from 150mm to 25mm or less. The flood season starts from June, and the monthly runoff from June to September accounts for 80% of annual runoff. In Shuangliao County, annual runoff depth is less than 10mm, due to low annual precipitation and flat topography with great soil sandy property and poor runoff generation conditions. The average temperature becomes negative from November annually, and the precipitation is converted to snow, leading to surface frozen, runoff terminated, rivers recharged by groundwater with small flow or even zero flow.

B. *Materials*

The experimental area was located in the middle of Dongliao River Basin, where the topography was low-lying from southeast to northwest, and a total of 36 grids were laid with the size of 15km×15km. According to different soil types along grid diagonal, 13 sampling points (see Fig.1) were arranged to monitor soil stable infiltrations at the depths of 30cm and 50cm, soil bulk density, mechanical composition, organic matter, total P, total N and soil moisture content at the depth of 10cm, 20cm and 30cm respectively.

Soil stable infiltration characteristics were measured by Guelph Permeameter 2008K1. The ring knife with the depth of 5cm, the diameter of 5cm and the volume of 100cm^3 was used to collect undisturbed soil for on-site measurement of soil bulk density. The soil moisture content and organic matter content were determined by drying method and thermal dilution method respectively. Mastersizer 2000 laser granulometer was applied to analyze soil particles. In accordance with the existing soil texture classification standard in China, soil particles were divided into three categories, i.e., clay (< 0.002mm), silt (0.002 ~ 0.05mm) and sand (0.05 ~ 1.0mm). The sulfuric acid-perchloric acid heating digestion method and semi-micro Kjeldahl method were used for determining the contents of total P and total N respectively. Except for soil stable infiltration characteristics and soil moisture content, the average value of 0 ~ 1.0m soil was taken as basic soil physical properties at various measuring points (see Table I).

TABLE I. SOIL STABLE INFILTRATION CHARACTERISTICS AND PHYSICAL PROPERTIES AT VARIOUS MEASURING POINTS

Sample	I30	I50	Bd	SM10	SM20	SM30	Om	Sd	St	Cc	P	N
S1	0.024	0.020	10.01	13.77	16.55	16.78	1.73	46.44	16.32	37.23	0.81	1.20
S2	0.039	0.030	13.20	8.92	11.00	12.52	0.62	48.65	26.53	24.82	0.87	1.44
S3	0.058	0.049	12.20	9.51	11.37	8.20	23.25	44.48	24.49	31.02	1.06	1.42
S4	0.021	0.021	14.36	10.36	9.63	13.34	13.38	17.79	38.77	43.44	0.76	2.08
S5	0.039	0.020	14.36	10.36	9.63	13.34	12.62	50.77	30.61	18.61	0.86	1.49
S6	0.000	0.001	13.43	14.09	11.07	18.13	1.50	28.16	40.81	31.02	0.90	1.51
S7	0.137	0.137	9.87	6.68	13.52	12.57	12.53	28.15	40.81	31.02	0.84	1.89
S8	0.003	0.156	11.53	8.86	11.45	12.72	11.51	34.27	34.70	31.02	1.00	0.94
S9	0.021	0.020	8.07	13.92	15.52	16.33	10.46	34.19	28.57	37.23	1.19	2.08
S10	0.039	0.041	18.01	5.09	7.24	7.05	11.42	71.27	16.32	12.41	0.74	1.16
S11	0.059	0.034	13.07	10.33	10.60	11.40	5.89	38.53	42.85	18.61	0.77	1.92
S12	0.068	0.009	12.78	7.52	10.23	11.61	17.16	52.81	28.57	18.61	0.68	1.82
S13	0.020	0.020	15.94	9.46	10.23	11.61	14.93	52.81	28.57	18.61	0.65	1.78

Note: I30 is the soil stable infiltration rate at the depth of 30cm (cm / min); I50 is the soil stable infiltration rate at the depth of 50cm (cm / min); Bd is soil bulk density (g/cm^3); SM10, SM20 and SM30 are soil moisture content at the depth of 10cm, 20cm and 30cm respectively (%); Om is the organic matter content (%); Sd, St and Cc are the contents of sand, silt and clay respectively (%); P and N indicate the contents of total phosphorus and total nitrogen respectively (g / kg). The data is not complete at measurement point S5, replaced by the value of adjacent measurement point S4.

C. *Methods*

Multifractal method is defined as a set consisting of infinite scaling exponents in the fractal structure. A spectrum function is used to describe the fractal characteristics at different levels and study the general characteristics starting from local system. Multifractal theory mainly consists of four parameters: mass index $\tau(q)$, generalized dimension

D(q), singularity index *α(q)* and dimension distribution function *f(q)*. Each parameter is calculated as follows[5-8]:

1) Mass index *τ(q)* If the research variable has multifractal characteristics, then for any given *q* value, mass index can be expressed as below:

$$\tau(q) = \lim_{\delta \to 0} \frac{\lg \sum_{i=1}^{n} P_i^q(\delta)}{\lg \delta} \tag{1}$$

$$P_i(\delta) = \mu_i \Big/ \sum_{i=1}^{n} \mu_i \tag{2}$$

In the equation, n stands for the number of intervals at the scale of δ; μ_i stands for the value of research variable in i-th (i = 1, 2, 3, ..., n) interval at the scale of δ ; $P_i(\delta)$ is the mass probability to describe the local distribution characteristics of research variable; q is the order of $P_i(\delta)$ statistical moment, which can be equal to any value. If $q>1$, high-value information is amplified, while if $q<-1$, low-value information is amplified. In this study, infiltration experiment was conducted in the study area with 36 grids sized by 15km×15km, where intervals with 5 kinds of scale were generated (δ=4/36, 9/36, 16/36, 25/36, 36/36).

2) Generalized dimension *D(q)* According to the relationship between *D(q)* and *τ(q)*, it can be known that:

$$D(q) = \begin{cases} \dfrac{1}{q-1} \lim_{\delta \to 0} \dfrac{\lg \sum_{i=1}^{n} P_i^q(\delta)}{\lg \delta} & q \neq 1 \\ \lim_{\delta \to 0} \dfrac{\sum_{i=1}^{n} P_i(\delta) \lg P_i(\delta)}{\lg \delta} & q = 1 \end{cases} \tag{3}$$

In the equation: when q=0, *D0* is box dimension, only considering whether the sampling point exists in the grid; when q=1, *D1* is information dimension, involving the number of sampling points in non-empty grid and revealing the scale change degree of pattern intensity as well as the non-uniform conditions of sampling point distribution; when q=2, *D2* is correlation dimension, reflecting the spatial correlation of sampling point changes with scales and revealing the spatial correlation degree of sampling points; if *D1* or *D2* is relatively small, long-distance variability will have a significant influence, while if *D1* or *D2* is relatively great, short－distance variability will have a significant influence; when $q \geqslant 0$, if *D(q)* decreases with q increasing, then research variable has multifractal characteristics.

3) The calculation equations of singularity index *α(q)* and dimension distribution function *f(q)* are shown as below:

$$\alpha(q) = \lim_{\delta \to 0} \frac{\sum_{i=1}^{n} \mu_i(q,\delta) \lg P_i(\delta)}{\lg \delta}, \quad f(q) = \lim_{\delta \to 0} \frac{\sum_{i=1}^{n} \mu_i(q,\delta) \lg \mu_i(q,\delta)}{\lg \delta} \tag{4}$$

In the equation: $\mu_i(q,\delta) = P_i^q(\delta) \Big/ \sum_{i=1}^{n} P_i^q(\delta)$ 。

The curves of *α(q)* and *f(q)* are called as multifractal spectrum. The larger the range of multifractal spectrum is, the stronger the spatial variability of variable will be. If multifractal spectrum deviates to the left, high-value information of variable will have a significant influence on spatial pattern. Contrarily, low-value information of variable will have a significant influence on spatial pattern.

III. RESULTS AND DISCUSSION

A. Correlation analysis between SSIC and influencing factors at point scale

The variability coefficient (CV) indicates the variability magnitude between research objects. The variability is weak, medium and strong, in case of CV≤0.1, 0.1<CV<1 and CV≥1 respectively. Table II gives the variability and correlation coefficients of soil stable infiltration characteristics and influencing factors at point scale. The variability coefficients of soil stable infiltration rate at the depths of 30cm and 50cm, clay content and organic matter content are the maximum, followed by those of soil water content at the depths of 10cm and 30cm, sand and silt content, while those of total P and N content, soil bulk density and soil water content at the depth of 20cm are the minimum. The soil stable infiltration rates at the depths of 30cm and 50cm show the medium and strong variability respectively, while all influence factors show medium variability. The soil stable infiltration rates at the depth of 30cm and 50cm have the most significant correlation to the soil moisture content at the depth of 10 cm. Therefore, the spatial variability's of soil stable infiltration rates at the depth of 30cm and 50cm at point scale might be caused by those of clay content, organic matter content and soil moisture content at the depth of 10 cm.

B. Spatial variability analysis of SSIC and influencing factors at multi-scale

In order to discuss whether soil stable infiltration characteristics and influencing factors have multifractal characteristics, *D(q)~q* curve (see Fig.2) and multifractal spectrum (see Fig.3) of soil stable infiltration characteristics and influencing factors in case of $-2 \leq q \leq 2$ were prepared. For clear calculation, Table IIIgives the specific *D(q)* values. It can be seen in Table III that in case of $q \geq 0$, *D(q)* of soil stable infiltration characteristics and organic matter content will be decreasing with q increasing, that of sand content will be constant basically, while those of other influencing factors will be increasing, i.e., soil infiltration characteristics and organic matter content have multifractal characteristics, while other influencing factors have single fractal characteristics. That is to say, the correlation of soil infiltration characteristics and soil physical properties at point scale will not be completely applicable at multi-scale.

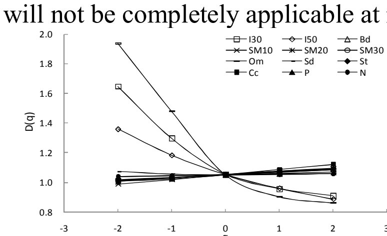

Figure 2. *D(q)~q* of soil stable infiltration characteristics and influencing factors.

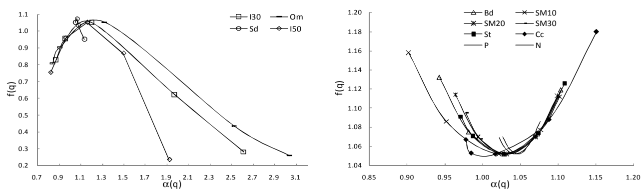

Figure 3. Multifractal spectra of soil stable infiltration characteristics and influencing factors.

According to the analysis on *D1* and *D2* in Table III, it can be seen that *D1* and *D2* of soil stable infiltration rates at the depth of 30cm and 50cm and organic matter content are the minimum, followed by those of soil moisture contents at the depth of 20cm and 30cm, total P and N contents and sand content, while those of soil moisture content at the depth of 10cm, soil bulk density, silt and clay content are the maximum. It is proven that the spatial variabilities of soil stable infiltration rates at the depth of 30cm and 50cm and organic matter content are mainly long-distance, while those of soil moisture content at the depth of 10cm, soil bulk density, silt and clay content are mainly short-distance. The multifractal spectrum widths ($\alpha_{max}(q)$-$\alpha_{min}(q)$) of soil stable infiltration rates at the depth of 30cm and 50cm and organic matter content are relatively great, indicating strong spatial variability. However, those of other parameters are less than 0.5, indicating weak spatial variability. The multifractal spectra of soil stable infiltration rates at the depth of 30cm and 50cm, soil moisture content at the depth of 30cm, total P content, silt, clay and sand content deviate to the left, indicating that the spatial variability is caused by high-value in the intervals with different scales. However, those of soil moisture contents at the depth of 10cm and 20cm, soil bulk density and total N content deviate to the right, indicating that the spatial variability is caused by low-value in the intervals with different scales.

C. Correlation analysis between SSIC and influencing factors at multi-scale

In order to determine the correlation between soil infiltration characteristics and influencing factors at multi-scale, the correlations of α_{I30} and α_{I50} to α_{Bd}, α_{SM10}, α_{SM20}, α_{SM30}, α_{Om}, α_{Sd}, α_{St}, α_{Cc}, α_P and α_N were analyzed respectively (see Table IV). It can be seen in Table IVthat the correlations between singularity indices of soil stable infiltration characteristics and influencing factors are relatively high, and those of α_{I30} and α_{I50} to α_N, α_{SM10}, α_{Bd} and α_{Om} are the most significant. Based on multifractal theory, it can be known that soil stable infiltration rates at the depth of 30cm and 50cm have the most significant correlation to soil moisture content at the depth of 10cm, soil bulk density, organic matter content and total N content. It is proven in single-scale (point scale) and multi-scale analysis that the influencing factors will have different influencing degrees on soil infiltration characteristics at different scales. The conversion model of soil infiltration characteristics should focus on the factors with significant influences on soil infiltration characteristics at different scales.

TABLE II. STATISTICS CHARACTERISTIC OF SOIL STABLE INFILTRATION CHARACTERISTICS AND INFLUENCING FACTORS

	Max.	Min.	Mean	Ave.	SD	CV	R_{I30}	R_{I50}
I30	0.137	0	0.039	0.041	0.036	0.873	/	/
I50	0.156	0.001	0.021	0.043	0.048	1.114	/	/
Bd	18.01	8.07	13.07	12.83	2.63	0.21	-0.20	-0.29
SM10	14.09	5.09	9.51	9.91	2.75	0.28	-0.55	-0.44
SM20	16.55	7.24	11.0	11.39	2.50	0.22	0.06	0.12
SM30	18.13	7.05	12.57	12.74	3.11	0.24	-0.37	-0.21
Om	23.25	0.62	11.51	10.54	6.60	0.63	0.30	0.18
Sd	71.27	17.79	44.48	42.18	13.96	0.33	0.01	-0.24
St	42.85	16.32	28.57	30.61	8.74	0.29	0.18	0.25
Cc	43.44	12.41	31.02	27.20	9.32	0.34	-0.18	0.13
P	1.19	0.65	0.84	0.86	0.15	0.18	-0.17	0.24
N	2.08	0.94	1.51	1.59	0.36	0.23	0.32	-0.32

Note: R_{I30} and R_{I50} are the correlation coefficients of I30 and I50 to influencing factors respectively. CV is the coefficient of variation; SD is the standard deviation.

TABLE III. MULTIFRACTAL PARAMETERS FOR SOIL STABLE INFILTRATION CHARACTERISTICS AND INFLUENCING FACTORS

	D0	*D1*	*D2*	$\alpha(0)$	$\alpha_{max}(q)$	$\alpha_{min}(q)$	$\alpha_{max}(q)$-$\alpha_{min}(q)$
I30	1.052	0.956	0.909	1.201	2.610	0.869	1.741
I50	1.052	0.959	0.886	1.159	1.921	0.820	1.101
Bd	1.052	1.073	1.090	1.029	1.104	0.943	0.161
SM10	1.052	1.077	1.092	1.022	1.102	0.902	0.200
SM20	1.052	1.070	1.086	1.032	1.099	0.964	0.135
SM30	1.052	1.070	1.085	1.032	1.099	0.978	0.121
Om	1.052	0.905	0.863	1.321	3.035	0.836	2.199
Sd	1.052	1.054	1.061	1.052	1.133	1.052	0.081
St	1.052	1.074	1.093	1.028	1.109	0.971	0.138
Cc	1.052	1.088	1.122	1.017	1.151	0.978	0.173
P	1.052	1.060	1.069	1.043	1.077	1.022	0.055
N	1.052	1.055	1.058	1.047	1.059	1.027	0.032

TABLE IV. CORRELATION BETWEEN SINGULARITY INDICES OF SOIL STABLE INFILTRATION CHARACTERISTICS AND INFLUENCING FACTORS

	α_{Bd}	α_{SM10}	α_{SM20}	α_{SM30}	α_{Om}	α_{Sd}	α_{St}	α_{Cc}	α_P	α_N
α_{I30}	-0.964	-0.979	-0.954	-0.933	0.992	0.898	-0.919	-0.812	-0.905	-0.984
α_{I50}	-0.983	-0.988	-0.976	-0.958	0.980	0.870	-0.945	-0.854	-0.939	-0.993

D. Establishment of the conversion model of SSIC

It can be seen in multifractal analysis that soil stable infiltration rates at the depth of 30cm and 50cm have the most significant correlation to soil moisture rate at the depth of 10cm, soil bulk density, organic matter content and total N content. Based on the conclusion in multifractal analysis, SPSS 18.0 software is used for nonlinear regression analysis to establish the conversion model of soil stable infiltration characteristics.

$$\begin{aligned} I_{30} &= -4.372 - 0.025N \cdot Om - 0.026N \cdot Bd - 0.155N \cdot SM10 \\ &- 0.003Om \cdot Bd + 0.004Om \cdot SM10 - 0.01Bd \cdot SM10 \\ &+ 2.209N + 0.045Om + 0.167Bd + 0.318SM10 \end{aligned} \quad (5)$$

$$
\begin{aligned}
I_{50} = {} & 4.711 - 0.028 N \cdot Om - 0.008 N \cdot Bd + 0.092 N \cdot SM10 \\
& + 0.01 Om \cdot Bd - 0.001 Om \cdot SM10 + 0.012 Bd \cdot SM10 \\
& - 0.607 N - 0.074 Om - 0.245 Bd - 0.322 SM10
\end{aligned} \tag{6}
$$

The above two equations are the conversion models of soil stable infiltration characteristics with the most significant influencing factors, of which the correlation coefficients are 0.996 and 0.958 respectively, above the significance level of 0.05. Fig. 4 gives the relationship between simulated value of soil stable infiltration characteristics calculated by soil conversion function and measured value. It can be seen in Fig. 4 that little calculation error occurs to the conversion model of soil stable infiltration characteristics at the depths of 30cm and 50cm, and RMSE is 0.048 和 0.050 respectively.

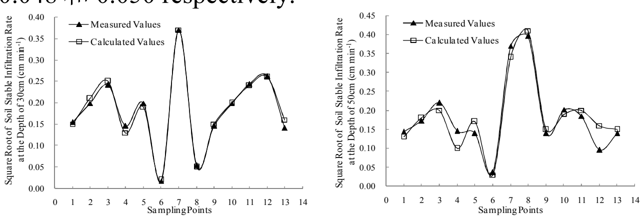

Figure 4. Relationship between square root of measured and calculated soil stable infiltration rates at the depths of 30cm (up) and 50cm (down).

IV. CONCLUSIONS

The soil infiltration in study area has multifractal properties. At point scale, soil stable infiltration rates at the depths of 30cm and 50cm show medium and strong variability respectively. They have the most significant correlation to soil moisture content at the depth of 10cm, of which spatial variabilities are mainly caused by those of clay content, organic matter content and soil moisture content at the depth of 10cm. At multi-scale, soil stable infiltration rates at the depths of 30cm and 50cm mainly show long-distance variability, of which the strong spatial variability is caused by high values in various intervals with different scales. Soil stable infiltration rates at the depths of 30cm and 50cm have the most significant correlation to water moisture content at the depth of 10cm, soil bulk density, organic matter content and total N content. Based on multifractal theory, the conversion models of soil stable infiltration characteristics at the depths of 30cm and 50cm are established with little calculation errors, of which the model correlation coefficients are 0.996 and 0.958 respectively, corresponding to RMSE of 0.048 and 0.050.

Based on multifractal theory, the conversion model of soil properties is established with a strong theoretical foundation and adaptability, providing the basis for larger-scale soil stable infiltration rate. However, only soil stable infiltration characteristics were discussed in this study, and non-stable infiltration characteristics remain to be further studied.

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