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Rainfall-Runoff Simulation Using Simulated Annealing Wavelet BP Neural Networks

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***Abstract*—Wavelet neural network is a powerful tool for rainfall-runoff (RR) prediction. In this essay, a neural network based on wavelet function was proposed. But due to the probability of reaching local minimum of WNN, an improved simulated annealing neural network SAWNN was used in comparison of the WNN, the SAWNN has the ability of reaching the global minimum by employing the disturbing function and is able to mapping non-linear relations. Results show that the SAWNN has ideal performance in RR simulation and has small training error. It also indicates that the training samples should contain as much samples in different condition as possible.**

Keywords- WNN; SAWNN; Training error; Rainfall-runoff BP; Simulated annealing

I. INTRODUCTION

Rainfall-runoff (RR) prediction is one of the most complicated processes in environmental modeling due to the tremendous spatial and temporal variability of topographical characteristics, rainfall patterns, and the number of parameters to be derived during the calibration. There emerge many methodologies in RR simulation since the Stanford Watershed Model was developed by Crawford and Linsley in 1966 [1]. However, accurate RR predictions largely depend on the long-term observation and recordings of precipitation and runoff, which can be costly especially when there are many observation locations in large watershed. The fast development in computing technology in recent years has made it possible to expand numerical simulation with high complexity; this resulted in an enhancement in watershed modeling. For different purpose, in addition to the RR simulation on a watershed, modelings aiming in water quality prediction, ecological system simulation are enjoying significant development as well. Not only these modeling perform in their areas but can be as components to other model as well. Nevertheless, physical based modeling are largely depends on the underlying information on different scales and many of them are to some extent have similarities in modeling mechanism and equations. Also, some models especially for water quality prediction may focus on microscopic interaction among substance regardless of many other relations on lager scales. As a result, integrating of different models or employing a model as a component somehow can be so far fetched or unrealistic. On the other hand, spatially distributed modeling relies on conservation equations of mass and momentum of watershed which further increase the load of computation.

While physical models are of importance in the understanding hydrological processes, there are many practical situations, such as evaporation simulation, flood prediction that concern accurate predictions. In such a situation, researchers may choose to implement a black box model instead of expending extra time and effort to develop a complicated model. For example, the ARMA (auto-regressive moving average) time series model, have been applied commonly in runoff simulation because of its easy development. Another successful black-box model is the ANN (Artificial Neural Network) that is expert at mapping non-linear relationship between inputs and outputs. AI (Artificial Intelligence) has been popular since 1990s and has been widely used in many areas. Ju et al., employ division-based BP neural networks in rainfall-runoff simulation [2]. The whole input data was divided into two independent clusters: flood period and non-flood period. The temporal improvement NN model was compared with the conceptual model and resulted in ideal numerical simulation. Moghaddamnia, A. et al. explored evaporation estimation based on ANN model and ANFIS (adaptive neuro-fuzzy inference system) techniques [3]. The result shows that ANN and ANFIS have better performances than the empirical formulas. Ahmad, S. et al. adopted ANN for prediction in Red River for peak flow, timing and shape of runoff hydrograph together with the employment of meteorological parameters including antecedent precipitation index, melt index etc [4]. The correlation between observed and simulated values of peak flow is high. ANN model can also be used for water quality simulation. Singh et al. has successfully carried out computation of DO and BOD concentrations using two ANN in the Gomti River water [5]. The identification, validation and test of the ANN are to some extent satisfactory. Chinh et al. has applied a feed-forward artificial neural network (FFANN) to model and simulate water levels in the main drainage canal [6]. The study indicated that the model can model the complex relationship between rainfall and water levels.

In addition to the successful application cases aforementioned, many improvements have been made to strengthen the performance of ANN. That includes integrating data preprocessing techniques with ANN, employing optimization algorithm to derive better weights of the nodes. Wu et al. introduced three data-preprocessing techniques, moving average (MA), singular spectrum analysis (SSA) etc. in the improvement of daily flows perdition [7]. The coupled ANN has the ability to get rid of the effect of white noise that may add error to the weight training. To improve on the drawbacks of the conventional optimal process, Chen et al. proposed a novel evolutionary artificial neural network (EANN) for time series forecasting [8]. The EANN employed genetic algorithm and the scaled conjugate gradient algorithm to identify and optimize the connection weights of neurons and the NN architecture. In general, NN model contains a number of interconnected mapping elements called neurons or nodes and is a data-driven, self-adaptive approach. Although many successful cases and improvements are mentioned, such as the hybrid time series analysis, optimization algorithms, traditional NN model can be improve further using other advanced theory and techniques. In this paper, a NN model hybrid wavelet function was proposed and was further improved using simulated annealing algorithm (by preventing local minimum) for global optimization. The SAWNN (simulated annealing wavelet back propagation neural network) was tested and compared with WNN model in the RR simulation.

II. METHODOLOGY

A. Wavelet Neural Networks

Neural network (NN) has the characteristics of fast learning, self-adaptive, good mapping ability. While by combing wavelet transformation and neural network, the traditional NN model can be improved for mapping rainfall-runoff. The wavelet neural network (WNN) is a kind of feed-forward neural network model. The linear wavelet neural network has its connection neurons between the hidden layer units and the output units by replacing original transformation functions (such as sigmoid function, radical basis functions) by basis functions including B-spline basis functions, wavelet basis functions and some other neuro-fuzzy basis function. The advantage of employing wavelet basis functions lies in their excellent performance in non-stationary signal analysis and nonlinear function modeling, which can provide higher availability of rates of convergence than ordinary NN [9]. The construction of WNN model can be described in figure 1.

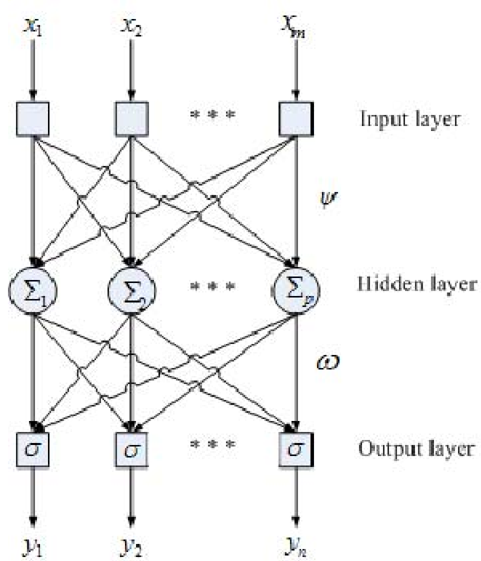

Figure 1. The feed-forward construction of WNN model

In terms of the wavelet transformation theory, assume that function $\Psi \in L^2$ follows the formula below:

$$C_\Psi = \int \frac{\left|\Psi_F(\omega)\right|^2}{|\omega|} d\omega < \infty \qquad (1)$$

Where: C_ψ was called the basic wavelet function, Ψ_F is the Fourier transform of Ψ.

While wavelets follow the form:

$$\Psi = \left\{ \Psi_i = |a_i|^{-\frac{1}{2}} \Psi\left(\frac{x-b_i}{a_i}\right), a_i, b_i \in R; i \in Z \right\} \qquad (2)$$

In equation (2), ψ_i is a set of child function from $\Psi(x)$. Where a_i is the scalar parameter while b_i is the translation parameter.

The output of the wavelet neural network y_i can be described as the formula below:

$$y_i = \sigma\left[\sum_{j=1}^{p} \omega_{ij} \left[\sum_{k=1}^{m} x_k(t) \psi\left(\frac{t-b_j}{a_j}\right)\right]\right], i = 1,2,\ldots,n \qquad (3)$$

In equation (3), σ is the sigmoid function and ψ is the wavelet bases function; x_k denotes the k^{th} input and y_i denotes the i^{th} output. While p represents the number of neurons, ω_{ij} is the weight between hidden layer and the out layer.

The sum square error function is listed below:

$$E = \frac{1}{2} \sum_{s=1}^{Sp} \sum_{i=1}^{n} \left(y_{sim,i} - y_i\right)^2 \qquad (4)$$

Where: *Sp* is the number of samples; *n* is the number of output. $y_{sim,i}$ is the mapping output and y_i is the observed output. The training goal is the reach the minimum of *E* by feed-back error to update the weight of neurons.

B. Simualted Annealing for Training Wavelet Network

Although BP-WNN has the characteristic of high convergence speed, it still can not avoid the premature convergence and may reach local minimum. The Simulated Annealing algorithm is a global optimization approach and the algorithm has good performance in searching global minimum. However, weight optimization by using simulated annealing algorithm may result in low convergence speed. Hence, by creating a hybrid of BP gradient searching algorithm and simulated annealing algorithm, both advantages in convergence speed and avoiding local minimum can be achieved. The detailed flow chart of SA-WNN and the steps can be as follows:

Step1: Initialize the annealing temperature T_0, set iteration $N=0$, iteration maximum N_{max} and minimun error E_{min}.

Step2: Initialize the structure of wavelet nerual network.

Step3: Start trainning using BP algorithm. Return the sum square error.

Step4: Check convergence. If $En<Emin$ or $N>Nmax$, end. If not, continue with step5.

Step5: Start annealing, update the weight ω by employing disturbing function. The formula is:

$$\omega^{N+1} = \omega^{N} + \alpha\Delta\omega \qquad (5)$$

Where: $\Delta\omega$ denotes the disturbing factor which follows the Cauchy distribution. α is the regulatory factor. The annealing function can be given as:

$$T_N = \frac{T_0}{1+\ln(N)} \qquad (6)$$

Check the $\Delta E = E^{N+1} - E^{N}$ with updated ω^{N+1}. If $\Delta E < 0$, accept ω^{N+1} and turn to Step 3. If not, compute the acceptance probability $p = \exp(\frac{\Delta E}{T_N})$. If $p \geq r$ (where r is a random number uniformly sampled between 0-1), accept ω^{N+1} and turn to Step 3. If not, repeat Step 5.

III. CASE STUDY

C. Data preparation

The appropriate input variables contain the correlation structure in the data. Rainfall-runoff process contains strong mapping relationship with the precipitation, runoff condition and the lag time. So the input data we adopted for testing are the precipitation data four days before current day and the runoff data three days before the current day. The constructed function of the runoff Q on current day t can be the following formula:

$$Q_t = f(Q_{t-1}, Q_{t-2}, Q_{t-3}, P_{t-1}, P_{t-2}, P_{t-3}, P_{t-4}) \qquad (7)$$

Where, Q_t is the runoff on the current day.

The sub-basin of the upstream area of the Danfeng hydrological station was selected as study case (figure 3). As is shown that there are totally 11 rainfall station uniformly scattered on the terrain map and 1 runoff station (Danfeng station) located in the downstream near the outlet of the sub-basin (figure 3). Data used in the simulation progress contains 11 year precipitation as well as the observed runoff from 1982 to 1985. The precipitation data is processed using reverse distance method to get the distance weighted rainfall series on the centroid of the sub-basin. All the data concerned in the training process is normalized between 0-1.

D. The number of hidden layers and nodes

Different number of hidden layers and nodes plays an important role in the network structure, which may directly affect the training result. A single hidden layer of neural

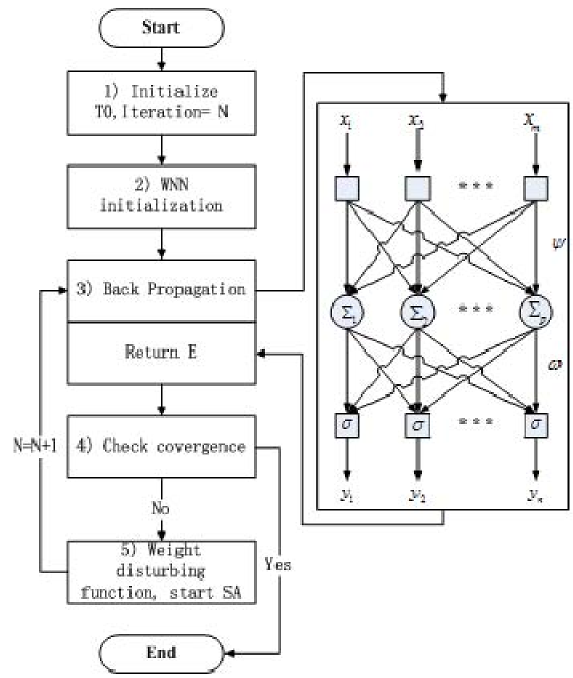

Figure 2. Flow chart of simulated annealing BP wavelet neural networks

networks is suitable for rainfall-runoff as many studies have referred in the literature. At present, there is no theory on identifying the node number. However, there still several empirical methods to follow that are:

$$m = \sqrt{n+l} + \alpha \qquad (8)$$

$$m = \log_2 n \qquad (9)$$

$$m = \sqrt{nl} \qquad (10)$$

Where: m is the number of nodes in hidden layer. n,l denotes the number of input data sets and the number of output set respectively. α is the constant between 1-10.

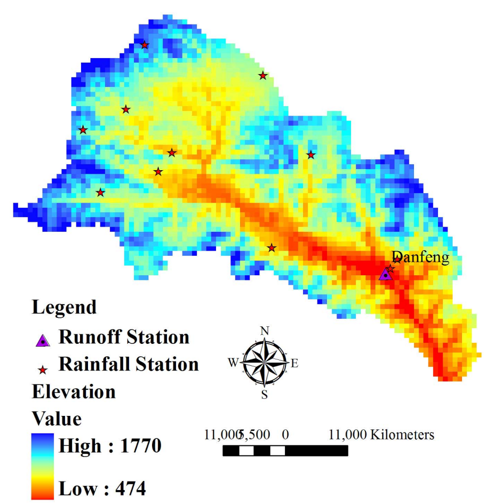

Figure 3. The sub-basin located on the upstream area of Danfeng hydrological gauge

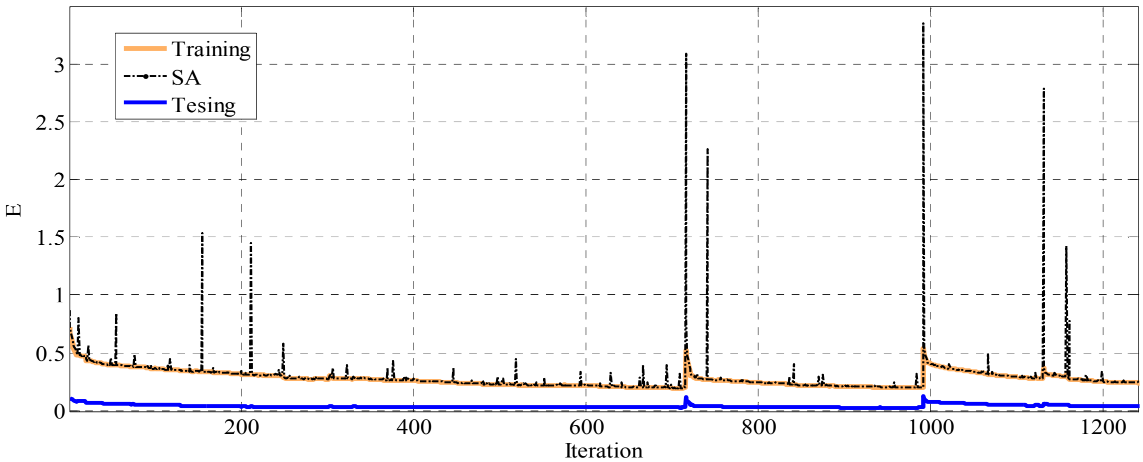

Figure 4. Training error, testing error and annealing temperature of SAWNN

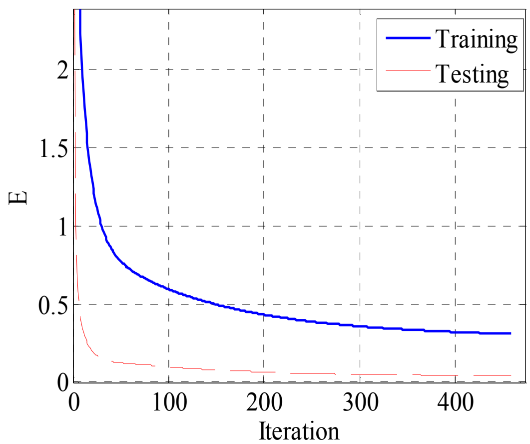

Figure 5. Training error and testing error of WNN

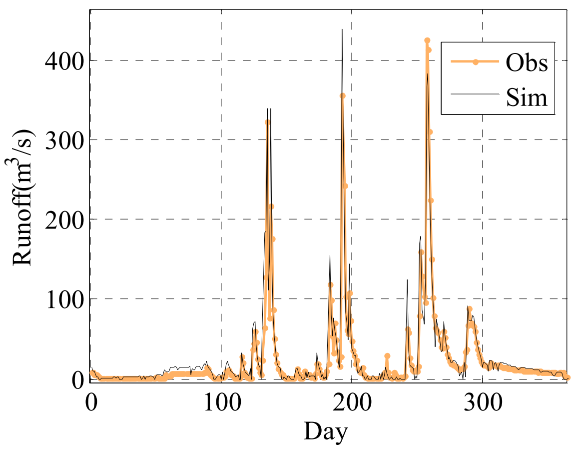

Figure 6. Testing results of SAWNN with minimum training error

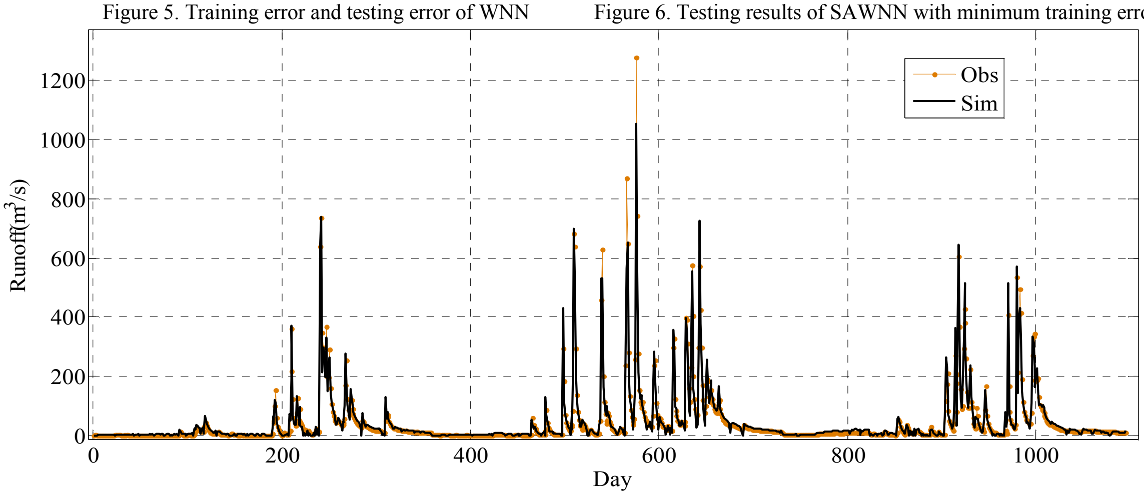

Figure 7. Training results of SAWNN with minimum training error

In this test, 7 sets of data are used for runoff simulation. After many times of pre-training have been carried out in terms of increasing number of hidden layer nodes, Then the more appropriate number of nodes is set to 10 for the later testing and the neural network structure can be as 7-10-1.

E. Application results

The data was divided into two groups that are data from 1982-1984 for weights training and the data of 1985 is used for performance test. Five independent training trails were carried out considering the randomness of simulation. Figure 4 and figure 5 show the convergence of training and testing error on increasing iteration. In figure 4, only BP algorithm was used during the weight training. The training error decreases greatly before iteration 200 however, changes slightly during the rest iteration. The minimum of the training error (Table I) is 0.3176 corresponding to the test error 0.0409. While figure 5 indicates the hybrid of simulated annealing and BP algorithm training, the algorithm compared with WNN has the ability of global searching by employing disturbing function. The minimum training error of SAWNN reaches 0.1921, which is nearly 10% lower than the WNN.

TABLE I. TEN INDEPENDENT TRAINING AND TESTING ERROR OF WNN AND SWANN

Trail	WNN		SAWNN	
	Emin	*Etmin*	*Emin*	*Etmin*
1	0.3176	0.0409	0.3033	0.0372
2	0.3612	0.0497	0.1921	0.0257
3	0.3446	0.0425	0.2067	0.0246
4	0.3203	0.0443	0.2816	0.0362
5	0.3988	0.0414	0.2118	0.0331

Figure 6 shows the training result of observation and simulation runoff in terms of minimum training error using SAWNN. While figure 7 displays the testing result in terms of minimum training error using SAWNN. As is implied that the training data contains the maximum runoff value, which is greater than the data in testing series. The training data comprise almost all the runoff conditions in the testing data and resulted in a better prediction. It is further indicates that the training data should contain as large samples as possible to carry more accurate prediction.

IV. CONCLUSION

The wavelet neural network is a powerful tool for prediction of rainfall-runoff. But the BP based WNN may has the probability of reaching local minimum during the weight training. To solve this problem, a global searching simulated annealing algorithm was introduced to improve the performance of weight training. Through analysis of the theory and the training and testing results, the SAWNN has the able to reach the global minimum which has better mapping ability than the WNN. The results show the generalization of SAWNN during non-linear learning when the condition of prediction is contained in the training data. In a word, SAWNN is suitable is the rainfall-runoff simulation and it produces ideal results than WNN.

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REFERENCES

[1]V.P. Singh, “Hydrologic Systems. Watershed Modeling”, vol. 2, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1989.

[2]Ju, Q., Z. Yu, et al., "Division-based rainfall-runoff simulations with BP neural networks and Xinanjiang model", Neurocomputing, vol. 72(13-15), pp. 2873-2883, 2009

[3]Moghaddamnia, A., M. Ghafari Gousheh, et al. “Evaporation estimation using artificial neural networks and adaptive neuro-fuzzy inference system techniques”, Advances in Water Resources, vol. 32(1), pp. 88-97, 2009.

[4]Ahmad, S. and S. P. Simonovic, "An artificial neural network model for generating hydrograph from hydro-meteorological parameters", Journal of Hydrology, vol. 315(1-4) pp. 236-251, 2005.

[5] Singh, K. P., A. Basant, et al., "Artificial neural network modeling of the river water quality--A case study", Ecological Modelling, vol. 220(6), pp. 888-895, 2009.

[6] Chinh, L. V., K. Hiramatsu, et al., "Estimation of water levels in a main drainage canal in a flat low-lying agricultural area using artificial neural network models", Agricultural Water Management, vol. 96(9), pp. 1332-1338, 2009.

[7]Wu, C. L., K. W. Chau, et al., "Methods to improve neural network performance in daily flows prediction", Journal of Hydrology, vol. 372(1-4), pp. 80-93, 2009.

[8] Chen, Y.-h. and F.-J. Chang, "Evolutionary artificial neural networks for hydrological systems forecasting", Journal of Hydrology, vol. 367(1-2), pp. 125-137, 2009.

[9] Changcheng, X., H. Xiyue, et al., “Wavelets Neural Network Based on Particle Swarm Optimization Algorithm for Fault Diagnosis”. Innovative Computing, Information and Contro l, 2006. ICICIC '06. First International Conference on, vol. 3, pp. 320-323, 2006.